Unit-V Complex Integration

Cauchy's theorem & Cauchy's-Integral formula:

1. Verify Cauchy's theorem by integrating e^{iz} along the boundary of the triangle with the vertices at the points 1+i, -1+I & -1-i. (12-13)
2. Evaluate the following integral using Cauchy's-Integral formula $\oint_C \frac{dz}{z^2(z^2-4)e^z}$ where C is the circle $|z| = 1$. (12-13)
3. Using Cauchy integral formula, evaluate $\oint_C \frac{\sin 2z}{(z+3)(z+1)^2} dz$, where C is$a\ rectangle\ with\ vertices\ at\ 3 \pm i, -2 \pm i$. (22-23)
4. Evaluate $\int_C \frac{(1+z)\sin z}{(2z-3)^2} dz$, where C is the circle $|z - i| = 2$ counter clockwise. (13-14)
5. Use Cauchy's integral formula to show that $\int_C \frac{e^{zt}}{z^2+1} dz = 2\pi i \sin t$ if $t > 0$ and C is the circle $|z| = 3$. (13-14)
6. State Cauchy's integral formula. Hence evaluate $\int_C \frac{2Z+1}{z^2+z} dz\ where\ C\ is\ |z| = {}^1/_2$. (14-15)
7. State & prove Cauchy-Integral formula. Also evaluate $\int_C \frac{1}{(Z^2+4)^2} dz = \frac{\pi}{16}$, where C is the circle $|z - i| = 2$. (17-18)
8. Evaluate by Cauchy-Integral formula $\oint_C \frac{z^2-2z}{(z+1)^2(z^2+4)} dz$ where C is the circle(i)$|z| = 3$. (15-16)(ii) $|z| = 10$. (17-18)
9. Using Cauchy Integral formula, Evaluate $\int_C \frac{\sin z}{(z^2+25)^2} dz$, where Cis the circle IzI=8. (18-19)
10. Evaluate $\int_C \frac{e^z}{z(1-z)^3} dz$ where C is (i)$|z| = \frac{1}{2}(ii)\ 2 < |z| < 3 (iii)|z| > 3$. (21-22)
11. Evaluate the following integral using Cauchy's-Integral formula $\oint_C \frac{4-3z}{z(z-1)(z-2)} dz$ where C is the circle $|z| = \frac{3}{2}$. (23-24)
12. Evaluate $\int_C \frac{e^z}{(z-1)(z-4)} dz$ where C is the circle $|z| = 2$, by using Cauchy's-Integral formula. (23-24)

Taylor's and Laurent series expansion:

1. Expand $f(z) = \frac{z}{(z-1)(2-z)}$ in Laurent series valid for $(i)|z - 1| > 1\ \ (ii) 0 < |z - 2| < 1$. (12-13)(23-24)
2. Find Taylor's and Laurent series with represent the function $\frac{z^2}{(z+2)(z+3)}$ when $(i)|z| < 2 (ii) 2 < |z| < 3 (iii)|z| > 3$.(21-22)
3. Expand $\frac{1}{z^2-3z+2}$ in the region $1 < |z| < 2$. (14-15)
4. Find the Laurent series for the function $f(z) = \frac{7z^2+9z-18}{z^3-9z}$, z is complex variable valid for the regions, $(i)\ 0 < |z| < 3 \qquad (ii)|z| > 3$. (15-16)
5. Expand$f(z) = \frac{7z-2}{z^3-z^2-2z}$ in the following region$(i) 0 < |z| < 1\ \ (ii) 1 < |z| < 2\ \ (iii)|z| > 2$. (12-13), (16-17)(22-23)
6. Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the regions, $(i)\ 1 < |z| < 2 \qquad (ii)\ 2 < |z|$. (18-19)

Residue Integration:

1. Determine the poles and residues at each pole for $f(z) = \frac{z-1}{(z+1)^2(z-2)}$ and hence evaluate $\oint_C f(z)\, dz$ where C is the circle $|z - i| = 2$. (13-14)
2. Determine the poles of the following function and residue at each pole; $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and hence evaluate $\int_C f(z)\, dz$, where C; $|z| = 3$. (14-15)
3. Find the poles (with its order) and residue at each poles of the following function; $f(z) = \frac{1-2z}{z(z-1)(z-2)^2}$. (16-17)
4. Find the Residue of $f(z) = \frac{z^3}{(z-1)^4(z-2)(z-3)}$ at its pole and hence evaluate $\int_C f(z)\, dz$, Where Cis the circle $|z| = {}^5/_2$.