Unit 01

Ordinary Different equations of Higher order

Diff equn :- An equation which contain diff cofficent $\left(\frac{d}{dx}, \frac{d^2}{dx^2}, \dots\dots \frac{d^n}{dx^n}\right)$ is called diff equn.

eg. $\frac{d^2y}{dx^2} + 2\frac{dy'}{dx} - 3y = \sin^2 x$

Order of a differential equn:
The order of a diff equn is the order of height ordered derivative contain in the diff equn.

Degree of a differential equn.
The degree of a differential eqn is the degree of height ordered derivative present in the differential eqn when it is made free from radical sign and fractional power.

eg. ① $\frac{d^2y}{dx^2} + \sqrt{1+\left(\frac{dy}{dx}\right)^2} = 0$

$$\left(\frac{d^2y}{dx^2}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$$

Order = 2, degree = 2

② $p = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$

$$p^2 = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^3}{\left(\frac{d^2y}{dx^2}\right)^2}$$

order = 2, degree = 2

Solution of differential eqwn

① ordinary differential eqn. of nth order with constant cofficient

The eqn of the from---

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1}}{dx^{n-2}} y + a_2 \frac{d^{n-2}}{dx^{n-2}} + \text{------} a_n y =$$

Where a_0, a_1, a_2 --- a_n are arbitrary constant and Q is fuction of x only or pure constant.

Solution.

Complete solution / General solution / particular soln

$$y = C.F + P.I$$

P.I: Particular Integration

C.F: Complementary function

Complementary function (C.F):

Replace

$$\frac{d}{dx} \to D, \quad \frac{d^2}{dx^2} \to D^2 \quad \frac{d^n}{dx^n} = D^n \text{ in eqn.}$$

$$(a_0 D^n + a_1 D^{n-1} + a_2 D^{n-2} + \text{-------} + a_n)\, y = Q \quad \text{——(2)}$$

Auxilary eqn.

Replace

$D \to m$ $\quad y = 1$ $\quad R.H.S = 0$ in eqn (2)

$$\boxed{a_0 m^n + a_1 m^{n-1} + a_2 m^{n-2} + \text{-------} + a_n = 0}$$

Case(I) when roots are real and distinct

let $m = m_1, m_2, m_3, m_4$

then $C.F = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x} + c_4 e^{m_4 x}$

Case(II) when roots are real & replaced

let $m = m_1, m_1, m_3, m_4$

then $C.F = c_1 e^{m_1 x} + x c_2 e^{m_1 x} + x c_4 e^{m_3 x}$

or let $m = m_1, m_1, m_1, m_1$

$C.F = c_1 e^{m_1 x} + x c_2 e^{m_1 x} + x^2 c_3 e^{m_1 x} + x^3 c_4 e^{m_1 x}$

Case(III) when roots are imaginary

let $m = \alpha \pm i\beta, \; a \pm ib$

$C.F = e^{\alpha x} (c_1 \cos \beta x + c_2 \sin \beta x) + e^{ax} (c_3 \cos bx + c_4 \sin bx)$

or

let $m = m_1, m_2, \alpha \pm i\beta$

then

$C.F = c_1 e^{m_1 x} + c_2 e^{m_2 x} + e^{\alpha x} (c_3 \cos \beta x + c_4 \sin \beta x)$

Case IV: when roots are imaginary & repeated

let $m = \alpha \pm i\beta, \; \alpha \pm i\beta$

$C.F = e^{\alpha x} [(c_1 + x c_2) \cos \beta x + (c_3 + x c_4) \sin \beta x]$

find the general $\cdot$ sol^n of

① $(2D+1)^2 y = 0$ where $D = d/dt$

② $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$

③ $\frac{d^3y}{dx^3} + \frac{2d^2y}{dx^2} + \frac{dy}{dx} = 0$

④ $(2D-1)^3 y = 0$ where $D = d/dt$

⑤ find the complementary function of $(D^2+a^2)y = 0$

⑥ $\cdot$ $\frac{d^2y}{dx^2} + 2\alpha\frac{dy}{dx} + y = 0$

Sol^n: (i) $(2D+1)^2 y = 0$

$y = C.F + P.I$

$PI = 0$

A.E, $(2m+1)^2 = 0$

$(2m+1)(2m+1) = 0$

$m = -1/2, -1/2$

$C.F = c_1 e^{-1/2 t} + \cancel{t} c_2 e^{-1/2 t}$

$y = \cdot(c_1 + tc_2)\, e^{-t/2}$

(ii) $(D^2 + D + 1)y = 0$

$y = CF + P.I$

$PI = 0$

A.E $m^2+m+1=0$

$m=-0.5\pm 0.866i$

$C.F = e^{-0.05x}(C_1\cos 0.866x + C_2\sin 0.866x)$

$y=e^{-0.5x}(c_1\cos 0.866x + c_2\sin 0.866x)$

(3) $(D^3+2D^2+D)y=0$

$y = C.F + P.I$

$P.I = 0$

A.E

$(m^3+2m^2+m)=0$

$m=0,-1,-1$

$C.F = c_1e^{0x}+c_2e^{-x}+xc_3e^{-x}$

$[y=c_1+c_2e^{-x}+xc_2e^{-x}]$

(4) $(2m-1)^3=0$

$m = 1/2, 1/2, -1/2$

$y = CF + P.I$

$PI = 0$

$CF = c_1e^{t/2}+tc_2e^{t/2}+t^2c_3e^{t/2}$

$[y=(c_1+tc_2+t^2c_3)e^{t/2}]$

(5) $(D^2+a^2)y=0$

AE

$m^2+a^2=0$

~~$m^2=$~~ . $m=\pm ai$

$m = 0\pm ai$

$CF = e^{0x}(c_1\cos x + c_2\sin ax)$

$[CF = c_1\cos ax + c_2\sin ax]$

(7) $(D^4 - a^4)y = 0$

AE $\quad m^4 - a^4 = 0$

$(m^2 - a^2)(m^2 + a^2) = 0$

$m = a, -a, \quad a \pm ai$

$C.F = c_1 e^{ax} + c_2 e^{-ax} + e^{ax}(c_3 \cos ax + c_4 \sin ax)$

$C.F = c_1 e^{ax} + c_2 e^{-ax} + c_4 \cos ax + c_4 a \sin ax$

(6) $D^2 + \cdot$

Q. Solve the diff eqn $\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$ where $R^2C - 4L$ and R_2, C, L are comfount.

Sol. $i = C.F + P.I$

$P.I = 0$

$\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right)I = 0$

AE

$m^2 + \frac{R}{L}m + \frac{1}{LC} = 0$

$$m = \frac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - 4\frac{1}{LC}}}{2}$$

$$m = \frac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - 4\frac{1}{\frac{4L}{R^2}}}}{2}$$

$$m = \frac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{R^2}{L^2}}}{2}$$

$m = -\frac{R}{2L} \pm 0$

$m = -\frac{R}{2L}, \ \frac{-R}{2L}$

$C.F = c_1 e^{-R/2L\, t} + t c_2 e^{-R/2L\, t}$

$i = c_1 e^{-Rt/2L} + t c_2 e^{-R/2L\, t}$

Solve the diff equation $\frac{d^2y}{dx^2} + y = 0$, given that $y(0) = 2$, $y(\pi/2) = -2$

$y = CF + PI$

$PI = 0$

$(D^2 + 1)y = 0$

A.E

$m^2 + 1 = 0$

$m^2 = -1$

$m = \pm i$

$C.F = e^{0x}(c_1 \cos x + c_2 \sin x)$

$CF = c_1 \cos x + c_2 \sin x$

$y = c_1 \cos x + c_2 \sin x$ —— (1)

(1) $y(0) = 2$

$y = 2$ where $x = 0$

$[c_1 = 2]$

$y = 2\cos x + c_2 \sin x$ —— (2)

(2) $y(\pi/2) = -2$

$y = -2$ when $x = \pi/2$

$[c_2 = -2]$

$[y = 2\cos x - 2\sin x]$

Particular Integral :(P.I)

Case(I) when R.H.S e^{ax} the $P.I = \frac{1}{f(D)} e^{ax}$

$$= \frac{1}{f(a)} e^{ax}$$

if $f(a) = 0$ case failed

then $P.I = \frac{x}{f'(D)} e^{ax}$

$$= \frac{x}{f'(a)} e^{ax}$$

Case(II) when R.H.S $\cos ax$ or $\sin ax$

$$PI = \frac{1}{f(D)} \sin ax \text{ or } \cos ax$$

putting $D^2 = -a^2$

Case(III) when R.H.S x^m or Algebric function then

$$P.I = \frac{1}{f(D)} x^n$$

Step.

① Take out the lowest degree term from $f(D)$ to make a first term unity.

② Take the factor $(1+\phi(D))$ or $(1-\phi(D))$

③ Take the factor in the numerator and make the from $(1+\phi D)^{-1}$ or $(1-\phi(D))^{-1}$

④ Expand it in ascending power of D, at the term of D^m.

(4) operate a x^m term by term.

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots$$

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots$$

Solve $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x + \cos x$

Solve.

$$(D^3 - 3D^2 + 4D - 2)y = e^x + \cos x$$

A.E

$$m^3 - 3m^2 + 4m - 2 = 0$$

$$m = 1, 1 \pm i$$

$$CF = c_1e^x + e^x(c_2\cos x + c_3\sin x)$$

$$P.I = \frac{1}{D^3 - 3D^2 + 4D - 2} e^x + \frac{1}{D^3 - 3D^2 + 4D - 2} \cos x$$

$D = a = 1$ $\qquad$ $D^2 = -a^2 = -1$

$$= \frac{1}{1 - 3 + 4 - 2} e^x + \frac{1}{-D + 3 + 4D - 2} \cos x$$

case failed

$$= \frac{x}{3D^2 - 6D + 4} e^x + \frac{1}{3D + 1} \cos x$$

$$= \frac{x}{3 - 6 + 4} e^x + \frac{3D - 1}{9D^2 - 1} \cos x$$

$$= xe^x + \frac{3D - 1}{-10} \cos x$$

$$= xe^x + \frac{3\sin x + \cos x}{10}$$

$y = C.F + P.I$

$y = c_1 e^x + e^x(c_2 \cos x + c_2 \sin x) + x e^x \frac{3\sin x + \cos x}{10}$

Q. $(D^2 - 4D + 5)y = e^{2x} + 3\cos(4x+3)$

$A.E = m^2 - 4m + 5 = 0$

$m = 2 \pm i$

$C.F = e^{2x}[c_1 \cos x + c_2 \sin x]$

$P.I = \frac{1}{D^2 - 4D + 5} e^{2x} + \frac{1}{D^2 - 4D + 5} 3\cos(4x+3)$

let $P_1 = \frac{1}{D^2 - 4D + 5} e^{2x}$

$D = a = 2$

$P_1 = \frac{1}{4 - 8 + 5} e^{2x}$

$P_1 = e^{2x}$

$P_2 = \frac{3\cos(4x+3)}{D^2 - 4D + 5}$

$D^2 = -a^2 = -16$

$P_2 = \frac{3\cos(4x+3)}{-16 - 4D + 5}$

$= 3 \frac{1}{-16 - 4D + 5} \cos(4x+3)$

$= \frac{3}{-4D - 11} \cos(4x+3)$

$= \frac{-3(4D - 11)}{16D^2 - 121} \cos(4x+3)$

$$= \frac{-3(4D-11)\cos(4x+3)}{16\times(-16)-121}$$

$$= \frac{-3[4D\cos(4x+3) - 11\cos(4x+3)}{-377}$$

$$= \frac{3}{377}[-16\sin x(4x+3) - 11\cos(4x+3)]$$ Ans

Solve:

$$\frac{d^2y}{dx^2} + 4y = \sin^2 x$$

$$y(0) = 0 \quad y'(0) = 0$$

Solve. $(D^2+4)y = \sin^2 x$

A.E = $m^2 + 4 = 0$

$m^2 = -4$

$m = \pm\sqrt{-4}$

$m = \pm i2$

$C.F = e^{0x}(c_1\cos 2x + c_2\sin 2x)$

$C.F = c_1\cos 2x + c_2\sin x)$

$$P.I = \frac{1}{D^2+4}\sin^2 x$$

$$= \frac{1}{D^2+4}\frac{1-\cos 2x}{2}$$

$$= \frac{1}{D^2+4}\frac{1}{2}e^{0x} - \frac{1}{D^2+4}\frac{1}{2}\cos 2x$$

$$= \frac{1}{4} - \frac{1}{2}\cdot\frac{1}{-4+4}\cos 2x$$

case faild

$$= \frac{1}{8} - \frac{1}{2}\frac{x}{2D}\cos 2x$$

$$= \frac{1}{8} - \frac{x}{4}\frac{\sin 2x}{2}$$

$$P.I = \frac{1}{8} - \frac{1}{8} x\sin 2x$$

$$y = C.F + P.I$$

$$y = c_1\cos 2x + c_2\sin 2x + \frac{1}{8} - \frac{1}{8}x\sin 2x \quad \text{--- (1)}$$

Q. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y + 37\sin 3x = 0$ and find the value of y when $x = \frac{\pi}{2}$ being given that $y = 3$, $\frac{dy}{dx} = 0$ when $x = 0$

Soln.. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = -37\sin 3x$

$(D^2 + 2D + 10)y = -37\sin 3x$

AE. $m^2 + 2m + 10 = 0$

$m = -1 \pm 3i$

$CF = e^{-x}[c_1\cos 3x + c_2\sin 3x]$

$P.I = \frac{1}{D^2 + 2D + 10}(-37\sin 3x]$

$= -37 \frac{1}{D^2 + 2D + 10} \sin 3x$

$D^2 = -a^2 = -9$

$= \frac{-37}{-9 + 2D + 10} \sin 3x$

$= \frac{-37}{2D + 1} \sin 3x$

$= \frac{-37\sin 3x}{2D + 1} \frac{(2D-1)}{(2D-1)}$

$= \frac{-37\sin 3x(2D-1)}{4D^2 - 1}$

$= \frac{-37\sin 3x(2D-1)}{-36-1} \Rightarrow \frac{-37\sin 3x(2D-1)}{-37}$

$P.I = 6\cos 3x - \sin 3x$

$y = C.F + P.I$

$y = e^{-x}[c_1\cos 3x + c_2\sin 3x] + 6\cos 3x - \sin 3x$ —(1)

(i) $y = 3$ where $x = 0$

$3 = c_1 + 6$

$[c_1 = -3]$

$y = e^{-x}(-3\cos 3x + c_2\sin 3x) + 6\cos 3x - \sin 3x$ —(2)

(2) $\frac{dy}{dx} = 0$ when $x = 0$

$$\frac{dy}{dx} = e^{-x}[9\sin 3x + 3c_2\cos 3x) - e^{-x}[-3\cos 3x + c_2\sin 3x] - 18\sin 3x - 3\cos 3x$$

$0 = 3c_2 + 3 - 3$

$[c_2 = 0]$

$y = -3e^{-x}\cos 3x + 6\cos 3x - \sin 3x$

y at $x = \frac{\pi}{2}$

$y = -3e^{-\pi/2}\cos\frac{3\pi}{2} + 6\cos\frac{3\pi}{2} - \sin\frac{3\pi}{2}$

$[y = 1]$ Ans

Q3: find the P.I of $\frac{d^2y}{dx^2} - y = x^2$

Sol^n: $P.I = \frac{1}{D^2-1} x^2$

$= \frac{1}{-(1-D^2)} x^2$

$= -[1-D^2]^{-1} x^2$

$= -[1+D^2] x^2$

$P.I = -[x^2+2]$ Ans

Q.

$(D^3-1)y = 3x^4 - 2x^3$

A.E $m^3 - 1 = 0$

$m = 1, -0.5 \pm 0.866 i$

or

$m = 1, -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

$C.F = C_1 e^x + e^{-x/2} [C_2 \cos \frac{\sqrt{3}}{2} x + C_3 \sin \frac{\sqrt{3}}{2} x)$

$P.I = \frac{1}{D^3-1} (3x^4 - 2x^3]$

$= \frac{1}{-1(1-D^3)} (3x^4 - 2x^3)$

$= -[1-D^3]^{-1} (3x^4 - 2x^3)$

$= -[1+D^3] (3x^4 - 2x^3)$

$P.I = -[3x^4 - 2x^3 + 72x - 12)$

$y = C.F + P.I$

$y = C_1 e^x + e^{-x/2} [c_2 \cos \frac{\sqrt{3}}{2} x + c_3 \sin \frac{\sqrt{3}}{2} x) - [3x^4 - 2x^3 + 72x - 12)$

Ans

Q. $(D^2 - 3D + 2)y = x^2 + 2x + 1$

Soln. A.E

$$m^2 - 3m + 2 = 0$$

$$m = 2, 1$$

$$C.F = C_1 e^{2x} + c_2 e^{x}$$

$$P.I = \frac{1}{D^2 - 3D + 2} (x^2 + 2x + 1)$$

$$= \frac{1}{2\left[1 + \frac{D^2 - 3D}{2}\right]} (x^2 + 2x + 1)$$

$$= \frac{1}{2}\left[1 + \left(\frac{D^2 - 3D}{2}\right)^{-1}\right] (x^2 + 2x + 1)$$

$$= \frac{1}{2}\left[1 - \left(\frac{D^2 - 3D}{2}\right) + \left(\frac{D^2 - 3D}{2}\right)^2\right] (x^2 + 2x + 1)$$

$$= \frac{1}{2}\left[1 - \left(\frac{D^2 - 3D}{2}\right) + \left(\frac{D^2 - 3D}{2}\right)^2\right] (x^2 + 2x + 1)$$

$$= \frac{1}{2}\left[1 - \frac{D^2}{2} + \frac{3D}{2} + \frac{1}{4} 9D^2)\right] (x^2 + 2x + 1)$$

$$= \frac{1}{2}\left[x^2 + 2x \cancel{+1} \cancel{+1} + 3(x+1) + \frac{9}{2}\right]$$

$$= \frac{1}{2}\left[x^2 + 2x + 3x + 3 + \frac{9}{2}\right]$$

$$= \frac{1}{2}\left[x^2 + 5x + \frac{15}{2}\right] \text{ Ans}$$

Q. $(D^3 - D^2 - 6D)y = 1 + x^2$

Sol^n. A.E $m^3 - m^2 - 6m = 0$

$m = 3, 0, -2$

$C.F = C_1 e^{3x} + C_2 e^{0x} + C_3 e^{-2x}$

$$P.I = \frac{1}{D^3 - D^2 - 6D}(1+x^2)$$

$$P.I = \frac{1}{-6D\left[1 + \frac{D^3 - D^2}{-6D}\right]}(1+x^2)$$

$$= \frac{1}{-6D\left[1 - \left(\frac{D^2 - D}{6}\right)\right]}(1+x^2)$$

$$= \frac{1}{-6D}\left[1 - \left(\frac{D^2 - D}{6}\right)\right]^{-1}(1+x^2)$$

$$= \frac{1}{-6D}\left[1 + \frac{D^2}{6} - \frac{D}{6} + \frac{1}{36}D^2\right](1+x^2)$$

$$= \frac{1}{-6D}\left[1 + x^2 + \frac{1}{3} - \frac{x}{3} + \frac{1}{18}\right]$$

$$= -\frac{1}{6D}\left[x^2 - \frac{x}{3} + \frac{25}{18}\right]$$

$$= -\frac{1}{6}\left[\frac{x^3}{3} - \frac{x^2}{6} + \frac{25}{18}x\right] \text{ Ans}$$

$y = C.F + P.I$

$$y = C_1 e^{3x} + C_2 e^{0x} + C_3 e^{-2x} + -\frac{1}{6}\left[\frac{x^3}{3} - \frac{x^2}{6} + \frac{25}{18}x\right]$$

HW

Q. $\frac{d^2y}{dx^2} + y = e^{2x} + \cosh x + x^3$

Q. $\frac{d^2x}{dt^2} + \frac{q}{b}(x-a) = 0 \quad a, b$

Solve: $\frac{d^2x}{dt^2} + \frac{q}{b}x - \frac{q}{b}a = 0$

$$\left(D^2 + \frac{q}{b}\right)x = \frac{q}{b}a$$

A.E $\quad m^2 + \frac{q}{b} = 0$

$$m^2 = -\frac{q}{b}$$

$$m = \pm i\sqrt{\frac{q}{b}}$$

$$C.F = c_1 \cos\sqrt{\frac{q}{b}}\,t + c_2 \sin\sqrt{q/b}\,t$$

$$P.I = \frac{1}{D^2 + \frac{q}{b}} \frac{qa}{b} e^{0t}$$

$$D = a = 0$$

$P.I = a$

$x = C.F + PI$

$$\left[\, x = c_1 \cos\sqrt{\frac{q}{b}}\,t + c_2 \sin\sqrt{\frac{q}{b}}\,t + a \,\right] \quad \underline{\text{Ans}}$$

Case(IV) when R.H.S $= e^{ax} V$

then $P.I = \frac{1}{f(D)} e^{ax} V$

$$= e^{ax} \frac{1}{f(D+a)} V$$

Note: where $V =$ cosax or sinax or x^m then using pirious method.

Q.13. Solve $(D^2 - 2D + 1)y = e^x \sin x$

A.E $\cancel{m^2 - 2D}$ $m^2 - 2m + 1 = 0$

$$[m = 1, 1]$$

$C.F = c_1 e^x + x c_2 e^x$

$$P.I = \frac{1}{D^2 - 2D + 1} e^x \sin x$$

$$= e^x \frac{1}{(D+1)^2 - 2(D+1) + 1} \sin x$$

$$= e^x \frac{1}{D^2 + 1 + 2D - 2D - 2 + 1} \sin x$$

$$= \frac{e^x}{D^2} \sin x$$

$D^2 = -a^2 = -1$

$$= \frac{e^x}{-1} \sin x$$

$P.I = -e^x \sin x$

$$\boxed{y = c_1 e^x + x c_2 e^x - e^x \sin x}$$

Q.13. Solve $(D^2 - 2D + 5)y = e^{2x} \cos x$

A.E $=$ $m^2 - 2m + 5 = 0$

$m = 1 \pm 2i$

C.F $= e^{x} [c_1 \cos 2x + c_2 \sin 2x]$

$$P.I = \frac{1}{D^2 - 2D + 5} e^{2x} \cos x$$

$$= \frac{e^{2x}}{(D+2)^2 - 2(D+2) + 5} \cos x$$

$$= \frac{e^{2x}}{D^2 + \cancel{4} + 4D - 2D \cancel{-4} + 5} \cos x$$

$$= \frac{e^{2x}}{D^2 + 2D + 5} \cos x$$

$D^2 = -a^2 = -1$

$$= \frac{e^{2x}}{-1 + 2D + 5} \cos x$$

$$= \frac{e^{2x}}{2D + 4} \cos x$$

$$= \frac{e^{2x}}{(2D+4)} \cos x \frac{(2D-4)}{(2D-4)}$$

$$= \frac{e^{2x} \cdot \cos x (2D-4)}{4D^2 - 16}$$

$$= \frac{e^{2x}\cos x\,(2D-4)}{-20}$$

$$= \frac{e^{2x}\,(2\sin x + 4\cos x)}{20} = \frac{e^{2x}(\sin x + 2\cos x)}{10}$$

$y = C.F + P.I$

$$y = e^{x}\,[c_1\cos 2x + c_2\sin 2x)\; e^{2x}\frac{(\sin x + 2\cos x)}{10}$$ Ans

Q.14. $\frac{d^2y}{dx^2} - \frac{2\,dy}{dx} + y = x^2 e^{3x}$

$(D^2 - 2D + 1)y = e^{3x}x^2$

A.E = $m^2 - 2D + 1 = 0$

$m = 1, 1$

$C.F = c_1 e^{x} + x c_2 e^{x}$

$$P.I = \frac{1}{D^2 - 2D + 1}\, e^{3x} x^2$$

$$= \frac{e^{3x}}{(D+3)^2 - 2(D+3) + 1}\, x^2$$

$$= \frac{e^{3x}}{D^2 + 9 + 6D - 2D - 6 + 1}\, x^2$$

$$= e^{3x}\frac{1}{D^2 + 4D + 4}\, x^2$$

$$= \frac{e^{3x}}{4} \frac{1}{1+\left(\frac{D^2+4D}{4}\right)} x^2$$

$$= \frac{e^{3x}}{4}\left[1+\frac{D^2+4D}{4}\right]^{-1} x^2$$

$$= \frac{e^{3x}}{4}\left[1-\left(\frac{D^2+4D}{4}\right)+\left(\frac{D^2+4D}{4}\right)^2\right] x^2$$

$$= \frac{e^{3x}}{4}\left[1-\frac{D^2}{4}-D+\frac{1}{16}16D^2\right] x^2$$

$$= \frac{e^{3x}}{4}\left[x^2-\frac{1}{2}-2x+2\right]$$

$$\left[P.I = \frac{e^{3x}}{4}\left[x^2-2x+\frac{3}{2}\right]\right]$$

$$y = C.F + PI$$

$$\left[y = c_1e^x + xc_2e^x + \frac{e^{3x}}{4}\left[x^2-2x+\frac{3}{2}\right]\right]$$ Ans

Q11: Solve. $(D^2-4D+4)y = 8x^2e^{2x}\sin 2x$

A.E $\quad m^2-4D+4=0$

$m = 2, 2.$

$C.F = c_1e^{2x} + xc_2e^{2x}$

$$P.I = \frac{1}{D^2 - 4D + 4} 8x^2 e^{2x} \sin 2x$$

$$P.I = \frac{e^{2x}}{(D+2)^2 - 4(D+2) + 4} 8x^2 \sin 2x$$

$$= \frac{e^{2x}}{D^2 + 4 + 4D - 4D - 8 + 4} 8x^2 \sin 2x$$

$$= \frac{e^{2x}}{D^2} 8x^2 \sin 2x$$

$$= \frac{8e^{2x}}{D^2} x^2 \sin 2x$$

$$= 8e^{2x} \frac{1}{D} \left[x^2 \left(-\frac{\cos 2x}{2} \right) - 2x \left(-\frac{\sin 2x}{4} \right) + 2 \left(\frac{\cos 2x}{8} \right) \right]$$

$$= 8e^{2x} \frac{1}{D} \left[-\frac{1}{2} (x^2 \cos 2x) + \frac{1}{2} (x \sin 2x) + \frac{1}{4} \cos 2x \right]$$

$$= 8e^{2x} \left[-\frac{1}{2} \left\{ x^2 \frac{\sin 2x}{2} - 2x \left(-\frac{\cos 2x}{4} \right) + 2 \left(-\frac{\sin 2x}{8} \right) \right\} + \frac{1}{2} \left(x \frac{-\cos 2x}{2} - \frac{-\sin 2x}{4} \right) + \frac{1}{4} \frac{\sin 2x}{2} \right]$$

$$= 8e^{2x} \left[-\frac{1}{4} x^2 \sin 2x - \frac{1}{4} x \cos 2x + \frac{1}{8} \sin 2x - \frac{1}{4} x \cos 2x + \frac{1}{8} \sin 2x + \frac{1}{8} \sin 2x \right]$$

$$= 8e^{2x} \left[-\frac{1}{4} x^2 \sin 2x - \frac{1}{2} x \cos 2x + \frac{3}{8} \sin 2x \right]$$

Q10. $(D^2 - 2D + 1)y = xe^x \sin x$

A.E $(m^2 - 2D + 1)y = 0$

$m^2 - 2m + 1 = 0$

$m = 1, 1$

$C.F = c_1 e^x + x c_2 e^x$

$P.I = \frac{1}{D^2 - 2D + 1} e^x x \sin x$

$= \frac{1}{D^2 - 2D} \frac{e^x}{(D+1)^2 - 2(D+1) + 1} x \sin x$

$= \frac{e^x}{D^2 + 1 + 2D - 2D - 2 + 1} x \sin x$

$= \frac{e^x}{D^2} x \sin x$

$= \frac{e^x}{D} [-x \cos x + \sin x]$

$= e^x [-x \sin x - (-\cos x) - \cos x]$

$P.I = e^x [-x \sin x - 2\cos x]$

$y = C.F + P.I$

$y = c_1 e^x + x c_2 e^x + e^x [-x \sin x - 2\cos x]$ Ans

Q15. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x^2 e^{-x} \cos x$

$(D^2 + 2D + 1)y = x^2 e^{-x} \cos x$

A E

$m^2 + 2m + 1 = 0$

$m = -1, -1$

$$C.F = c_1 e^{-x} + \cancel{\ } \, x c_2 e^{-x}$$

$$P.I = \frac{1}{D^2+2D+1} e^{-x} x^2 \cos x$$

$$= \frac{e^{-x}}{(D-1)^2 + 2(D-1) + 1} x^2 \cos x$$

$$= \frac{e^{-x}}{D^2 + \cancel{1} - \cancel{2D} + 2D - \cancel{2} + 1} x^2 \cos x$$

$$= \frac{e^{-x}}{D^2} x^2 \cos x$$

$$= \frac{e^{-x}}{D} [$$

Q18. A body executes damped force vibration give by the equation $\frac{d^2x}{dt^2} + 2K\frac{dx}{dt} + b^2x = e^{-Kt}\sin\omega t$ solve the equation for both the cases, when $\omega^2 \neq b^2 - K^2$ and $\omega^2 = b^2 - K^2$

Solve $(D^2 + 2KD + b^2)x = e^{-Kt}\sin\omega t$

A.E $m^2 + 2Km + b^2 = 0$

$$m = \frac{-2K \pm \sqrt{4K^2 - 4b^2}}{2}$$

$$m = \frac{-2K \pm 2\sqrt{K^2 - b^2}}{2}$$

$$m = -K \pm \sqrt{-(b^2 - K^2)}$$

$$m = -K \pm i\sqrt{b^2 - K^2}$$

$$C.F = e^{-Kt}\left(c_1 \cos\sqrt{b^2-K^2}\,t + c_2 \sin\sqrt{b^2-K^2}\,t\right)$$

$$P.I = \frac{1}{D^2 + 2KD + b^2} e^{-Kt}\sin\omega t$$

$$= e^{-Kt}\frac{1}{(D-K)^2 + 2K(D-K) + b^2}\sin\omega t$$

$$= e^{-Kt}\frac{1}{D^2 + K^2 \cancel{- 2KD} \cancel{+ 2KD} - 2K^2 + b^2}\sin\omega t$$

$$= e^{-Kt}\frac{1}{D^2 + b^2 - K^2}\sin\omega t$$

$$D^2 = -a^2 = -\omega^2$$

$$P.I = e^{-Kt}\frac{1}{-\omega^2-K^2+b^2}\sin\omega t$$

case (i) when $\cancel{\omega^2 \neq b^2-K^2}$

$$x = C.F + P.I$$

$$x = e^{-Kt}\left(c_1\cos\sqrt{b^2-K^2}\,t + c_2\sin\sqrt{b^2-K^2}\,t\right) + e^{-Kt}\frac{1}{-\omega^2-(b^2-K^2)}\sin\omega t$$

case (ii) when $\omega^2 = b^2-K^2$

$$\therefore CF = e^{-Kt}(c_1\cos\omega t + c_2\sin\omega t)$$

$$P.I = \frac{e^{-Kt}}{-\omega^2+\omega^2}\sin\omega t$$

~~case fail~~

$x = C.F + PI$

$$= e^{-Kt}\frac{t}{2D}\sin\omega t$$

$$= e^{-Kt}\,t\left(\frac{-\cos\omega t}{2\omega}\right)$$

Q16: Solve $(D^2-2D+4)y = e^x\cos x + \sin x\cos 3x$

solve:

$$(D^2-2D+4)y = e^x\cos x$$
$$(D^2-2D+4)y = \sin x\cos 3x$$

$$A.E = m^2-2m+4=0$$
$$m = 1\pm 1.73i$$
$$m = 1\pm\sqrt{3}i$$
$$C.F = e^x(c_1\cos\sqrt{3}x + c_2\sin\sqrt{3}x)$$

Now. $$P.I = \frac{1}{D^2-2D+4}e^x\cos x$$

$$P.I = \frac{e^x}{(D+1)^2 - 2(D+1) + 4} \cos x$$

$$= \frac{e^x}{D^2 + 1 + \cancel{2D} - \cancel{2D} + 2 + 4} \cos x$$

$$= \frac{e^x}{D^2 + 3} \cos x$$

$$D^2 = -a^2 = -1$$

$$= \frac{e^x}{-1+3} \cos x$$

$$P.I_1 = \frac{e^x \cos x}{2}$$

Now $P.I_2 = \frac{1}{D^2 - 2D + 4} \cancel{\sin x \cos 3x}\ 2\cos 3x \sin x$

$$= \frac{1}{2(D^2 - 2D + 4)} \cancel{2\sin x \cos 3x}\ 2\cos 3x \sin x$$

$$= \frac{1}{2(D^2 - 2D + 4)} \sin 4x - \sin 2x$$

$$= \frac{1}{2(D^2 - 2D + 4)} \sin 4x - \frac{1}{2(D^2 - 2D + 4)} \sin 2x$$

$$D^2 = -a^2 = -16 \qquad D^2 = -a^2 = -4$$

$$= \frac{1}{2(-16 - 2D + 4)} \sin 4x - \frac{1}{2(-4 - 2D + 4)} \sin 2x$$

$$= \frac{1}{2[-2D - 12)} \sin 4x - \frac{1}{2[-2D]} \sin 2x$$

Case(V) When R.H.S $= x^m \sin ax$ or $x^m \cos x$

then $P.I = \frac{1}{f(D)} x^m \sin ax$ or $\frac{1}{f(D)} x^m \cos ax$

Note $e^{iax} = \cos ax + i \sin ax$

$P.I = I.P \frac{1}{f(D)} x^m e^{iax}$ or $R.P \frac{1}{f(D)} x^m e^{iax}$

using case(IV)

Q.17. Solve $(D^2 - 2D + 1)y = x \sin x$

Solv.

A.E $\quad m^2 - 2m + 1 = 0$

$m = 1, 1$

$C.F = c_1 e^x + x c_2 e^x$

$P.I = \frac{1}{D^2 - 2D + 1} x \sin x$ $\quad [\because e^{ix} = \cos x + i \sin x]$

$= I.P$ of $\frac{1}{D^2 - 2D + 1} x e^{ix}$

$=$ " $e^{ix} \frac{1}{(D+i)^2 - 2(D+i) + 1} x$

$=$ " $e^{ix} \frac{1}{D^2 - 1 + 2iD - 2D - 2i + 1} x$

$=$ " $e^{ix} \frac{1}{D^2 + (i-1)2D - 2i} x$

$$= \frac{e^{ix}}{-2i}\left[\frac{1}{1-\frac{D^2}{2i}-\frac{(i-1)D}{2i}}\right]x$$

$$= \text{"} \frac{e^{ix}}{-2i}\left[1-\left(\frac{D^2}{2i}+\frac{(i+1)D}{i}\right)\right]^{-1}x$$

$$= \text{"} \frac{e^{ix}}{-2i}\left[1+\frac{(i-1)D}{i}\right]x$$

$$= \text{"} \frac{e^{ix}}{-2i}\left[x+\frac{i-1}{i}\right] \Rightarrow \text{"} \frac{e^{ix}}{-2i}\left[x+\frac{i+i^2}{i}\right].$$

$$= \text{"} \frac{e^{ix}}{-2i}\left[x+1+i\right]$$

$$= \frac{ie^{ix}}{2}\left[x+1+i\right]$$

$$= \text{"} \frac{i(\cos x+i\sin x)}{2}\left[x+1+i\right]$$

$$= \text{"} \frac{1}{2}\left[ix\cos x+i\cos x-\cos x-x\sin x-\sin x-i\sin x\right]$$

$$P.I = \frac{1}{2}\left[x\cos x+\cos x-\sin x\right]$$

$$y = C.F + P.I$$

$$y = c_1e^x + xc_2e^x + \frac{1}{2}\left[x\cos x+\cos x-\sin x\right]$$

Ans

Solve $(D^2+1)y = z\sin z$

A.E $m^2+1=0$

$m = 0 \pm i$

$C.F = e^{0z}[(c_1\cos z + c_2\sin z)]$

$P.I = \frac{1}{D^2+1} z\sin z$

$= I.P \text{ of } \frac{1}{D^2+1} ze^{iz}$

$= " \; e^{iz} \frac{1}{(D+i)^2+1} z$

$= " \; e^{iz} \frac{1}{D^2+i^2+2iD+1} z$

$= " \; \frac{e^{iz}}{2iD} \left[\frac{1}{1+\frac{D}{2i}}\right] z$

$\because e^{iz} = \cos z + i\sin z$

$= " \; \frac{e^{iz}}{2iD}\left[1+\frac{D}{2i}\right]^{-1} z$

$= " \; \frac{e^{iz}}{2iD}\left[1-\frac{D}{2i}\right] z$

$= " \; \frac{e^{iz}}{2iD}\left[z-\frac{1}{2i}\right]$

$= " \; \frac{e^{iz}}{2iD}\left[\frac{z^2}{2}-\frac{1}{2i}z\right]$

$= " \; \frac{e^{iz}}{4i}\left[z^2-\frac{z}{i}\right]$

$= " \; \frac{ie^{iz}}{-4}[z^2+iz]$

$= " \; \frac{i(\cos z + i\sin z)}{-4}[z^2+iz]$

$= " \; \frac{-1}{4}[iz^2\cos z - z\cos z - z^2\sin z - iz\sin z]$

$P.I = \frac{-1}{4}[z^2\cos z - z\sin z]$

(Homogenous differential equation
Or Homogeneous Leaner equation.)

The equation of the form

$$x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + a_2 x^{n-2} \frac{d^{n-2}}{dx^{n-2}} + \cdots + a_n y = f(x) \quad \text{--- (1)}$$

where a_1, a_2, a_3 ------ a_n are constant is called Euler's - Cauchy's Homogeneous diff^{n} equation.

Step for Sol^n :-

Step (1) putting

$$x\frac{dy}{dx} = D$$

$$x^2 \frac{d^2}{dx^2} = D(D-1)$$

$$x^3 \frac{d^3}{dx^3} = D(D-1)(D-2)$$

put $x = e^z$ and $z = \log x$ in R.H.S where $D = \frac{d}{dz}$

Step (2) putting value in equation (1)

(3) Using the method of constant cofecient and solve the differential equation.

(2) Legendre Homogenous diff equ^{n}

$$(ax+b)^n \frac{d^n y}{dx^n} + a_1 (ax+b)^{n-1} \frac{d^{n-1}}{dx^{n-1}} + a_2 (ax+b)^{n-2} \frac{d^{n-2}}{dx^{n-2}} + \cdots + a_n y = f(x)$$

putting $ax+b = e^z$, $z = \log(ax+b)$ in R.H.S

$$(ax+b)\frac{d}{dx} = aD$$

$$(ax+b)^2\frac{d^2}{dx^2} = a^2 D(D-1)$$

$$(ax+b)^3\frac{d^3}{dx^3} = a^3 D(D-1)(D-2)$$

Q1. Solve $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x+\frac{1}{x}\right)$

Above diff equation is Homogenous diff equn.

let $x = e^z$, $z = \log x$ in R.H.S

and $x\frac{d}{dx} = D$

$$x^2\frac{d^2}{dx^2} = D(D-1) = D^2 - D$$

$$x^3\frac{d^3}{dx^3} = D(D-1)(D-2) = D^3 - 3D^2 + 2D$$

$$(D^3-3D^2+2D)y + 2(D^2-D)y + 2y = 10\left(e^z + \frac{1}{e^z}\right)$$

$$(D^3 - 3D^2 + 2D + 2D^2 - 2D + 2)y = 10(e^z + e^{-z})$$

$$(D^3 - D^2 + 2)y = 10(e^z + e^{-z})$$

A.E $\cancel{D^3 - D^2}$ $M^3 - M^2 + 2 = 0$

$$M = -1 , 1 \pm i$$

$$C.F = C_1e^{-z} + e^z(C_2\cos z + C_3\sin z)$$

$\pm ai$ $\quad$ $c_1\cos ax + c_2\sin ax$

$$P.I = 10\left[\frac{1}{D^3-D^2+2}e^{z} + \frac{1}{D^3-D^2+2}e^{-z}\right]$$

$$= 10\left[\frac{1}{1-1+2}e^{z} + \frac{1}{-1-1+2}e^{-z}\right]$$

Case fail

$$= 10\left[\frac{e^z}{2} + \frac{z\cdot e^{-z}}{3D^2-2D}\right]$$

$$= 10\left[\frac{e^z}{2} + \frac{z}{5}e^{-z}\right]$$

$$P.I = 5e^{z} + 2ze^{-z}$$

$y = C.F + P.I$

$$y = \frac{c_1}{e^z} + e^z(c_2\cos z + c_3\sin z) + 5e^z + \frac{2z}{e^z}$$

$$y = \frac{c_1}{x} + x\left[c_2\cos(\log x) + c_3\sin(\log x) + 5x + \frac{2\log x}{x}\right]$$

Q. solve $\left(\frac{d}{dx} + \frac{1}{x}\right)^2 y = x^{-4}$

solve $\left(\frac{d^2}{dx^2} + \frac{1}{x^2} + 2\frac{d}{dx}\frac{1}{x}\right)y = x^{-4}$

$$x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + y = x^{-4}$$

(5) Solve $(2x+3)^2 \frac{d^2y}{dx^2} - 2(2x+3)\frac{dy}{dx} - 12y = 6x$

putting $(2x+3) = e^z$ and $z = \log(2x+3)$ in P.4.S

$$2x = e^z - 3$$

$$x = \frac{1}{2}(e^z - 3)$$

$$(2x+3)\frac{d}{dx} = 2D$$

$$(2x+3)^2 \frac{d^2}{dx^2} = \cancel{4(D} \; 4D(D-1)$$

$$\Rightarrow [4D^2 - 4D - 4D - 12]y = 6\,\frac{1}{2}(e^z - 3)$$

$$(4D^2 - 8D - 12)y = 3e^z - 9$$

A.E $\quad 4M^2 - 8M - 12 = 0$

$$M = 3, -1$$

$\cancel{C.F = G}$ G $\quad C.F = c_1 e^{3z} + c_2 e^{-z}$

$$P.I = \frac{1}{4D^2 - 8D - 12}(3e^z - 9)$$

$$P.I = \frac{3e^z}{4D^2 - 8D - 12} - \frac{9e^0}{4D^2 - 8D - 12}$$

$$D = a = 1 \qquad\qquad D = a = 0$$

$$P.I = \frac{3e^z}{4 - 8 - 12} - \frac{9e^0}{-12}$$

$$P.I = \frac{3e^z}{-16} + \frac{3}{4}$$

$y = C.F + P.I$

$y = c_1 e^{3z} + c_2 e^{-z} - \frac{3}{16} e^{z} + \frac{3}{4}$

$$\left[y = c_1 (2x+3)^3 + \frac{c}{(2x+3)} - \frac{3}{16}(2x+3) + \frac{3}{4} \right] \text{ Ans}$$

(6) Solve $(x+1)^2 \frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} = (2x+3)(2x+4)$

putting $x+1 = e^z$ $\quad z = \log(x+1)$

$x = e^z - 1$

$(x+1)\frac{d}{dx} = D$

$(x+1)^2 \frac{d}{dx} = D^2 - D$

$\Rightarrow$ ~~$(D^2-D)y + Dy$~~ $=$

$[(D^2-D)+D]y = [(2e^z-2+3)(2e^z-2+4)]$

$= (2e^z+1)(2e^z+2)$

$D^2y = 4e^{2z} + 6e^z + 2$

A.E $\quad m^2 = 0$

$m = 0, 0$

$C.F = c_1 + z c_2$

$P.I = \frac{1}{D^2} 4e^{2z} + \frac{1}{D^2} 6e^z + \frac{1}{D^2} 2e^0$

$D = a = 2 \qquad D = a = 1$

$P.I = e^{2z} + 6e^z + \frac{1}{D} 2z$

$= e^{2z} + 6e^z + \frac{2z^2}{2}$

$= e^{2z} + 6e^z + z^2$

$P.I = e^{2z} + 6e^{z} + z^2$

$y = C.F + P.I$

$y = c_1 + zc_2 + e^{2z} + 6e^{z} + z^2$

$\left[y = c_1 + \log(x+1) + (x+1)^2 + 6(x+1) + \log(x+1)^2 \right]$ Ans

Q7. Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(x+1)$

putting $(1+x) = e^{z}$ $\quad z = \log(x+1)$

$x = e^{z} - 1$

$(1+x)\frac{d}{dx} = D$

$(1+x)^2 \frac{d^2}{dx^2} = D^2 - D$

$\Rightarrow \quad (D^2 \cancel{-D} \cancel{+D} + 1)y = 4\cos z$

$(D^2+1)y = 4\cos z$

A.E $\quad m^2 + 1 = 0$

$m = 0 \pm i$

$C.F = e^{0z}[c_1\cos z + c_2\sin z$

$P.I = \frac{1}{D^2+1} 4\cos z$

$D^2 = a^2 = -1$

$= \frac{4\cos z}{-1+1}$ case fail

$$= 4 \frac{z}{2D} \cos z$$

$$= \cancel{4\frac{z}{4D^2}} \quad 4\frac{z}{2D^2} \times D \cos z$$

$$= \frac{2zD\cos z}{-1}$$

$$= +2z\sin z$$

$$P.I = 2z\sin z$$

$$y = C.F + P.I$$

$$y = e^{0z}[c_1\cos z + c_2\sin z) + 2z\sin z$$

$$\left[\, y = c_1\cos(\log(x+1)\ c_2\sin\log(x+1) + 2\log(x+1)\sin\log(x+1))\right]$$

Q $$x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x + \log x$$

putting $x = e^z \qquad z = \log x$

$$x\frac{d}{dx} = D$$

$$x^2\frac{d^2}{dx^2} = D^2 - D$$

$$x^3\frac{d^3}{dx^3} = D(D-1)(D-2)$$
$$= D^3 - 3D^2 + 2D$$

$$[\cancel{D^3 - 3D^2 + 2D - D^2 - D},$$

$$[D^3 - 3D^2 + 2D + 3D^2 - 3D + D + 1]\,y = e^z + z$$

$$[D^3 + 1]\,y = e^z + z$$

A.E $\quad m^3 + 1 = 0$

$$m = -1,\ \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

$$C.F = c_1e^{-z} + e^{-1/2z}\left(c_2\cos\frac{\sqrt{3}}{2}z + c_3\sin\frac{\sqrt{3}}{2}z\right)$$

$$P.I = \frac{1}{D^3+1} e^z + \frac{1}{D^3+1} z$$

$$D = a = 1$$

$$= \frac{1}{1+1} e^z + \cancel{\frac{z}{[D^3+1]}} \frac{1}{[1+D^3]} \cancel{z}$$

$$= \frac{e^z}{2} + [1+D^3]^{-1} z$$

$$= \frac{e^z}{2} + [1-D^3] z$$

$$P.I = \frac{e^z}{2} + z + 0$$

$$y = C.F + P.I$$

$$y = c_1 e^{-z} + c_2 e^{-1/2 z} \left[c_2 \cos \frac{\sqrt{3}}{2} z + c_3 \sin \frac{\sqrt{3}}{2} z \right]$$

$$\left[y = \frac{c_1}{x} + c_2 \sqrt{x} \left[c_2 \cos \frac{\sqrt{3}}{2} \log x + c_3 \sin \frac{\sqrt{3}}{2} \log x \right] \right] \text{Ans}$$

(2) $$x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} - 12y = x^3 \log x$$

putting $x = e^z$ $\quad z = \log x$

$$x \frac{d}{dx} = D$$

$$x^2 \frac{d^2}{dx^2} = D^2 - D$$

$$[D^2 - D + 2D - 12] y = e^{3z} z$$

$$[D^2 + D - 12] y = z e^{3z}$$

A.E $\quad M^2 + M - 12 = 0$

$$M = 3, -4$$

$$C.F = C_1 e^{3z} + C_2 e^{-4z}$$

$$P.I = \frac{1}{D^2 + D - 12} e^{3z} z$$

$$= e^{3z} \frac{1}{(D+3)^2 + (D+3) - 12} z$$

$$= e^{3z} \frac{1}{D^2 + \cancel{9} + 6D + D + \cancel{3} - \cancel{12}} z$$

$$= \frac{e^{3z}}{D^2 + 7D} z$$

$$= \frac{e^{3z}}{7D\left[1 + \frac{D^2}{7D}\right]} z$$

$$= \frac{e^{3z}}{7D\left[1 + \frac{D}{7}\right]} z$$

$$= \frac{e^{3z}}{7D} \left[1 + \frac{D}{7}\right]^{-1} z$$

$$= \frac{e^{3z}}{7D} \left[1 - \frac{D}{7}\right] z$$

$$= \frac{e^{3z}}{7D} \left[z - \frac{1}{7}\right]$$

$$P.I = \frac{e^{3z}}{7} \left[\frac{z^2}{2} - \frac{1}{7} z\right]$$

$$y = C.F + P.I$$

$$y = c_1 e^{3z} + c_2 e^{-4z} + \frac{e^{3z}}{7} \left[\frac{z^2}{2} - \frac{1}{7} z\right]$$

$$\boxed{y = c_1 x^3 + \frac{c_2}{x^4} + \frac{x^3}{7}\left[\frac{(\log x)^2}{2} - \frac{1}{7}\log x\right]}$$ **Ans**

3) $(x^2D^2 - xD + 4)y = \cos(\log x) + x\sin\log x$

putting $x = e^z$ $z = \log x$

$x\frac{d}{dy} = D$

$x^2\frac{d^2}{dy^2} = D^2 - D$

$[D^2 - \cancel{D} - D + 4]y = \cos z + x\sin z$

$\cancel{[D^2 + 4]y = \cos z + x\sin z}$

$\cancel{A \cdot E \quad m^2 + 4 = 0}$

$\cancel{m = 0 \pm 2i}$

$\cancel{C \cdot F = C}$

$[D^2 - 2D + 4]y = \cos z + \overset{z}{\cancel{x}}\sin z$

$A \cdot E \quad m^2 - 2m + 4 = 0$

$m = 1 \pm \sqrt{3}i$

$C \cdot F = \cancel{C_1}\, e^z [C_1 \cos\sqrt{3}z + C_2 \sin\sqrt{3}z]$

$P \cdot I = \frac{1}{D^2 - 2D + 4}\cos z + \frac{1}{D^2 - 2D + 4}\overset{z}{\cancel{e}}\sin z$

$D^2 = -a^2 = -1$

$P \cdot I = \frac{1}{-1 - 2D + 4}\cos z + \frac{e^z}{(D+1)^2 - 2(D+1) + 4}\sin z$

$= \frac{1}{3 - 2D}\cos z + \frac{e^z}{D^2 + 1 + \cancel{2D} - \cancel{2D} - 2 + 4}\sin z$

$$= \cancel{\cos}\ \frac{(3+2D)\cos z}{9-4D^2} + \frac{e^z}{D^2+3}\sin z$$

$$= \frac{(3+2D)\cos z}{9-4(-1)} + \frac{e^z}{-1+3}\sin z$$

$$= \frac{(3+2D)\cos z}{13} + \frac{e^z}{2}\sin z$$

$$= \frac{3\cos z + 2(-\sin z)}{13} + \frac{e^z}{2}\sin z$$

$$P.I = \frac{3\cos z - 2\sin z}{13} + \frac{e^z}{2}\sin z$$

$y = C.F + P.I$

$$y = e^z[c_1\cos\sqrt{3}z + c_2\sin\sqrt{3}z) + \frac{3\cos z - 2\sin z}{13} + \frac{e^z}{2}\sin z$$

$$y = x[c_1\cos\sqrt{3}\log x + c_2\sin\sqrt{3}\log x) + \frac{3\cos\log x - 2\sin\log x}{13} + \frac{x}{2}\sin\log x$$

Ans

(4) $\frac{d^3y}{dx^3} - \frac{4}{x}\frac{d^2y}{dx^2} + \frac{5}{x^2}\frac{dy}{dx} - \frac{2y}{x^3} = 1$

$x^3\frac{d^3y}{dx^3} - 4x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} - 2y = x^3$

putting $x = e^z$ $\quad z = \log x$

$x\frac{d}{dx} = D$

$x^2\frac{d^2}{dx^2} = D^2 - D$

$x^3\frac{d^3}{dx^3} = D^3 - 3D^2 + 2D$

$[D^3 - 3D^2 + 2D - 4D^2 + 4D + 5D - 2]y = e^{3z}$

$[D^3 - 7D^2 + 11D - 2]y = e^{3z}$

A.E $\quad m^3 - 7m^2 + 11m - 2 = 0$

$m = 4.79, 0.20, 2$

$C.F = c_1e^{4.79z} + c_2e^{0.20z} + c_3e^{2z}$

$P.I = \frac{1}{D^3 - 7D^2 + 11D - 2}e^{3z}$

$D = a = 3$

$P.I = \frac{1}{27 - 63 + 33 - 2}e^{3z}$

$P.I = \frac{e^{3z}}{5}$

$y = C.F + P.I$

$y = c_1e^{4.79z} + c_2e^{0.20z} + c_3e^{2z} + \frac{e^{3z}}{5}$

$\left[y = c_1x^{4.79} + c_2x^{0.20} + c_3x^2 + \frac{x^3}{5} \right]$ Ans

H.W

$$(1+2x)^2\frac{d^2y}{dx^2} - 6(1+2x)\frac{dy}{dx} + 16y = 8(1+2x)^2$$

$$(x+a)^2\frac{d^2y}{dx^2} - 4(x+a)\frac{dy}{dx} + 6y = x$$

solve putting $(1+2x) = e^z$ $\quad z = \log(1+2x)$

$$2x = e^z - 1$$

$$x = \frac{1}{2}(e^z - 1)$$

$$(1+2x)\frac{d}{dx} = 2D$$

$$(1+2x)^2\frac{d^2}{dx^2} = 4D(D-1)$$
$$= 4D^2 - 4D$$

$$[4D^2 - 4D - 12D + 16]y = 8e^{2z}$$
$$[4D^2 - 16D + 16]y = 8e^{2z}$$
$$[D^2 - 4D + 4]y = 2e^{2z}$$

A.E $\quad M^2 - 4M + 4 = 0$

$$M = 2, 2$$

$$C.F = c_1e^{2z} + zc_2e^{2z}$$

$$P.I = \frac{1}{D^2 - 4D + 4} 2e^{2z}$$

$$= 2\frac{e^{2z}}{D^2 - 4D + 4}$$

$$= \quad D = a = 2$$

$$= 2\frac{e^{2z}}{4 - 4\times 2 + 4}$$

$$= 2\frac{e^{2z}}{0}$$ case fail

$$= 2\frac{z}{2D - 4} \Rightarrow \frac{2z}{2D-4} \times \frac{2D+4}{2D+4}$$

$$\frac{2z(2D+4)}{4D^2 - 16}$$

$$= 2z(2D+4)$$

Q. $(x+a)^2 \frac{d^2y}{dx^2} - 4(x+a)\frac{dy}{dx} + 6y = x$

Sol. put $x+a = e^z$ $\quad$ $\therefore z = \log(x+a)$

$x = e^z - a$

$(x+a)\frac{dy}{dx} = D$

$(x+a)^2 \frac{d^2}{dx^2} = D(D-1) = D^2 - D$

$[D^2 - D - 4D + 6]y = e^z - a$

$[D^2 - 5D + 6]y = e^z - a$

A.E $\quad m^2 - 5m + 6 = 0$

$m = 3, 2$

$C.F = c_1 e^{3z} + c_2 e^{2z}$

$P.I = \frac{1}{D^2 - 5D + 6} e^z - \frac{1}{D^2 - 5D + 6} a e^0$

$D = a = 1 \qquad D = a = 0$

$= \frac{1}{1 - 5 + 6} e^z - \frac{1}{6} a$

$P.I = \frac{e^z}{2} - \frac{a}{6}$

$\therefore y = C.F + P.I$

$y = c_1 e^{3z} + c_2 e^{2z} + \frac{e^z}{2} - \frac{a}{6}$

$\left[y = c_1 (x+a)^3 + c_2 (x+a)^2 + \frac{(x+a)}{2} - \frac{a}{6} \right]$ Ans

Sol^n of Second order Diff^n eq^n:-

1. Standard form second order diff^n equation is given by

$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$ where P, Q, R are functions of x or constant.

1) Method of Variation of parameter method (VOPM):-

let $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$

Second order Diff^n equation.

Step ① make the cofficient of $\frac{d^2y}{dx^2}$ as 1

Step ② find out the part of C.F and let them u and v

Step ③ let $y = Au + Bv$ is sol^n of the given differential eq^n ①

where $A = -\int \frac{Rv}{W}\,dx + C_1$

$$B = \int \frac{Ru}{W}\,dx + C_2$$

(wronskian) $W = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix} = uv' - u'v$

Variation of Parameter Method

1) find the particular soln of $\frac{d^2y}{dx^2} + a^2y = \sec ax$

<u>solve</u> A.E $m^2 + a^2 = 0$

$m^2 = -a^2$

$m = 0 \pm ai$

$C.F = e^{0x}[c_1 \cos ax + c_2 \sin ax)$

let $u = \cos ax \qquad v = \sin ax$

$u' = -a\sin ax \qquad v' = a\cos ax$

$$w = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix}$$

$$w = \begin{vmatrix} \cos ax & \sin ax \\ -a\sin ax & a\cos ax \end{vmatrix}$$

$w = a\cos^2 ax + a\sin^2 ax$

$\boxed{w = a}$

let $y = AU + BV$

$A = \cdot\int \frac{RV}{w} dx + c_1$

$= -\int \frac{\sec ax \sin ax}{a} dx + c_1$

$= \frac{1}{a}\int \frac{-\sin ax}{\cos ax} dx + c_1$

$= \frac{1}{a^2}\int \frac{-a\sin ax}{\cos ax} dx + C$

$$\boxed{A = \frac{1}{a^2}\log \cos ax + c_1}$$

$$B = \int \frac{Ru}{\omega}\,dx + c_2$$

$$= \int \frac{\sec ax \cos ax}{a}\,dx + c_2$$

$$= \frac{1}{a}\int 1\,dx + c_2$$

$$\boxed{B = \frac{x}{a} + c_2}$$

$$\boxed{y = c_1 \cos ax + c_2 \sin ax + \frac{\cos ax}{a^2}\log \cos ax + \frac{x \sin ax}{a}}$$ Ans

Q17. $\frac{d^2y}{dx^2} + a^2 y = \cos ax$

$(D^2 + a^2)y = 0$

A.E $\quad M^2 + a^2 = 0$

$M = 0 \pm ai$

$\boxed{C.F = c_1 \cos ax + c_2 \sin ax}$

let $\quad u = \cos ax \qquad v = \sin ax$

$u' = -a \sin ax \qquad v' = a \cos ax$

$$W = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix}$$

$$W = \begin{vmatrix} \cos ax & \sin ax \\ -a \sin ax & a \cos ax \end{vmatrix}$$

$$\boxed{W = a}$$

let $y = Au + Bv$ —— ②
is sol^n of eq^n ①

$$A = -\int \frac{Rv}{\omega} dx + c_1$$

$$= -\int \frac{\operatorname{cosec} ax}{a} \sin ax \, dx + c_1$$

$$= \frac{-1}{a} \int \frac{\sin ax}{\sin ax} dx + c_1$$

$$= -\frac{1}{a} \int dx + c_1$$

$$A = -\frac{x}{a} + c_1$$

$$B = \int \frac{Ru}{\omega} dx + c_2$$

$$B = \int \frac{\operatorname{cosec} ax}{a} \cos ax \, dx + c_2$$

$$= \frac{1}{a^2} \int \frac{a \cos ax}{\sin ax} dx + c_2$$

$$= \frac{1}{a^2} \log(\sin ax) + c_2$$

$$y = c_1 \cos ax + c_2 \sin ax - \frac{x \cos ax}{a} + \frac{\sin ax}{a} \log(\sin ax)$$

(2) $\frac{d^2y}{dx^2} + y = \tan x$

A.E $D^2+1 = \tan x$

$M^2+1=0$

$m = 0 \pm i$

$C.F = e^{0x}(C_1 \cos x + C_2 \sin x)$

let $u = \cos x$ $\quad v = \sin x$

$u' = -\sin x$ $\quad v' = \cos x$

$$W = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix}$$

$$= \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$= \cos^2 x + \sin^2 x$

$\boxed{W = 1}$

$y = Au + Bv$ —— (2) is sol^n of eq^n (1)

$A = -\int \frac{Rv}{W} dx + C_1$

$A = -\int \tan x \sin x \, dx + C_1$

$= -\int \frac{\sin x}{\cos x} \sin x \, dx + C_1$

$= -\int \frac{\sin^2 x}{\cos x} dx + C_1$

$= -\int \frac{1-\cos^2 x}{\cos x} dx + C_1$

$= -[\int \sec x \, dx - \int \cos x \, dx] + C_1$

$$= -\left[\log(\sec x + \tan x)\right] - \sin x \,] + c_2$$

$$A = \sin x - \log(\sec x + \tan x) + c_1$$

$$B = \int \frac{RU}{w}\, dx + c_2$$

$$= \int \tan x \cos x \, dx + c_2$$

$$= \int \frac{\sin x}{\cos x} \cos x \, dx + c_2$$

$$= \int \sin x \, dx + c_2$$

$$B = -\cos x + c_2$$

$$y = AU + BV$$

$$y = \sin x \cos x - \cos x \log(\sec x + \tan x) + c_1 \cos x - \sin x \cos x + c_2 \sin x$$

4) Solve $(D^2-3D+2)y = \sin(e^{-x})$

Sol:- $m^2-3m+2=0$

$m = 2, 1$

$C.F = \cancel{c_1\cos 2x}\ c_1e^{2x}+c_2e^{x}$

let $u = e^{2x}$ $\quad v = e^{x}$

$u' = 2e^{2x}$ $\quad v' = e^{x}$.

$$W = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix}$$

$$= \begin{vmatrix} e^{2x} & e^{x} \\ 2e^{2x} & e^{x} \end{vmatrix}$$

$$= e^{3x} - 2e^{3x}$$

$$\left[W = -e^{3x} \right]$$

$y = Au + Bv$

$= Ae^{2x} + Be^{x}$

$$A = -\int \frac{Ru}{W}\,dx + c_1$$

$$= -\int \frac{(\sin e^{-x})\, e^{2x}}{-e^{3x}}\,dx + c_1$$

$$= \int \frac{\sin e^{-x}}{e^{x}}\,dx + c_1$$

$$= \int e^{-x} \sin e^{-x}\,dx + c_1$$

let $\cancel{e^{-x}=t}$ $\quad e^{-x} = t$

$\cancel{-e^{-x}=dt}$ $\quad -e^{-x}\,dx = dt$

$$= -\cancel{\int t \sin(t)\,dt} + c_1$$

$$= -\int \sin(t)\,dt + c_1$$

$$= \cos(t) + c_1$$

$$\boxed{A = \cos(e^{-x}) + c_1}$$

$$B = \int \frac{RV}{w}\,dx + c_2$$

$$= \int \frac{\sin e^{-x}}{-e^{2x}}\,dx + c_2$$

$$= -\int e^{-2x} \sin e^{-x}\,dx + c_2$$

let $\cancel{e^{-2x}}$ $e^{-x} = t$

$-e^{-x}dx = dt$

$$= -\int e^{-x} - e^{-x} \sin e^{-x}\,dx + c_2$$

$$= \int t \sin(t)\,dt + c_2$$

$$= -t\cos t - \sin(t) + c_2$$

$$B = (t\cos t - \sin t) + c_2$$

$$y = c_1 e^{2x} + c_2 e^{x} + e^{2x}[\cos e^{-x}] + e^{x}(t\cos t - \sin t)$$

$$= c_1 e^{2x} + c_2 e^{x} + e^{2x}[\cos e^{-x}] + e^{x}[e^{-x}\cos e^{-x} - \sin e^{-x}]$$

5) Solve the following differential equation by the method of VOPM. $y_2 - 3y_1 + 2y = e^{2x} + x^2$

Soln:
$$\frac{d^2y}{dx^2} - \frac{3dy}{dx} + 2y = e^{2x} + x^2$$

$$(D^2 - 3D + 2)y = e^{2x} + x^2$$

$$M^2 - 3M + 2 = 0$$

$$M = 1, 2$$

$$C.F = c_1 e^{x} + c_2 e^{2x}$$

let $u = e^{x}$ $\quad$ $v = e^{2x}$

$u' = e^{x}$ $\quad$ $v' = 2e^{2x}$

$$w = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix} = \begin{vmatrix} e^{x} & e^{2x} \\ e^{x} & 2e^{2x} \end{vmatrix}$$

$$= 2e^{3x} - e^{3x}$$

$$\boxed{w = e^{3x}}$$

let $y = Au + Bv$

$= Ae^{x} + Be^{2x}$

$$B = \int \frac{Ru}{w}\, dx + c_2$$

$$= \int \frac{(e^{2x} + x^2)e^{x}}{e^{3x}}\, dx + c_2$$

$$= \int \frac{(e^{2x} + x^2)}{e^{2x}}\, dx + c_2$$

$$= \theta\left[\int 1\,dx + \int e^{-2x}x^2\,dx\right] + c_2$$

$$= \theta\left[x + \frac{x^2e^{-2x}}{-2} - \frac{2x\,e^{-2x}}{-2\times(-2)} + 2\frac{e^{-2x}}{-8}\right] + c_2$$

$$= \theta\left[x - \frac{x^2e^{-2x}}{2} - \frac{x\,e^{-2x}}{2} - \frac{e^{-2x}}{4}\right] + c_1$$

$$B = x + \frac{x^2e^{-2x}}{2} + \frac{x\,e^{-2x}}{2} + \frac{e^{-2x}}{4} + c_2$$

$$A = -\int\frac{RV}{W} + c_1$$

$$= -\int\frac{(e^{2x}+x^2)\,e^{2x}}{e^{3x}}\,dx + c_1$$

$$= -\int\frac{(e^{2x}+x^2)}{e^{x}}\,dx + c_1$$

$$= -\left[\int e^{x}\,dx + \int e^{-x}x^2\,dx\right] + c_1$$

$$= -\left[e^{x} + \frac{x^2e^{-x}}{-1} - \frac{2x\,e^{-x}}{1} + \frac{2e^{-x}}{-1}\right] + c_1$$

$$= -\left[e^{x} - x^2e^{-x} - 2xe^{-x} - 2e^{-x}\right] + c_1$$

$$A = -e^{x} + x^2e^{-x} + 2xe^{-x} + 2e^{-x} + c_1$$

$$y = AU + BV$$

$$y = e^{x}\left[x + x^2\frac{e^{-2x}}{2} + x\frac{e^{-2x}}{2} + \frac{e^{-2x}}{4}\right] + e^{2x}\left[-e^{x} + x^2e^{-x}\right.$$

$$\left.+ 2xe^{-x} + 2e^{-x}\right] + c_1e^{x} + c_2e^{x}$$ Ans

Q6:

Solve

$$C.f = c_1 e^{x} + c_2 e^{-x}$$

let $u = e^{x}$ $\quad v = e^{-x}$

$u' = e^{x}$ $\quad v' = -e^{-x}$

$$w = -1-1 = -2$$

$$A = -\int \frac{RV}{w}\,dx + c_1$$

$$= -\int \frac{\cancel{2}}{(1+e^{x})} \frac{e^{-x}}{(-\cancel{2})}\,dx + c_1$$

$$= \int \frac{e^{-x}}{1+e^{x}}\,dx + c_1$$

$$= \int \frac{e^{-x}e^{-x}}{e^{-x}+1}\,dx + c_1$$

putting $\quad e^{-x}+1 = t$

$e^{-x} = t-1$

$-e^{-x}dx = dt$

$e^{-x}dx = -dt$

$$= -\int \frac{(t-1)}{t}\,dt + c_1$$

$$= -\left[\int 1\,dt - \int \frac{1}{t}\,dt\right] + c_1$$

$$= -[t - \log t] + c_1$$

$$= -t + \log t + c_1$$

$$A = -[e^{-x}+1] + \log(e^{-x}+1) + c_1$$

$$B = \int \frac{RU}{W} + c_2$$

$$= \int \frac{2}{(1+e^x)(-2)} e^x \, dx + c_2$$

$$= -\int \frac{e^x}{1+e^x} dx + c_2$$

$$\boxed{B = -\log(1+e^x) + c_2}$$

$$y = c_1 e^x + c_2 e^{-x} + e^x\left[-(e^{-x}+1) + \log(e^{-x}+1)\right] + e^{-x}\left[-\log(1+e^x)\right]$$ Ans

(7) use method of variation of Parameter to find the particular integral of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}$

Solⁿ $(D^2 - 3D + 2)y = \frac{e^x}{1+e^x}$

$m^2 - 3m + 2 = 0$

$m = 1, 2$

$C.F = c_1 e^x + c_2 e^{2x}$

let $u = e^x$ $\quad v = e^{2x}$

$u' = e^x$ $\quad v' = 2e^{2x}$

$W = 2e^{3x} - e^{3x}$

$$\boxed{W = e^{3x}}$$

$$B = \int \frac{Ru}{W} dx + C_2$$

$$= \int \frac{e^x \quad e^x}{(1+e^x) e^{3x}} dx + C_2$$

$$= \int \frac{1}{e^x(1+e^x)} dx + C_2$$

$$= \cancel{+\int \frac{-e^{x} e^{x}}{1+e^{x}} dx + C_2}$$

$$= \int \frac{e^{-x}}{1+e^x} dx + C_2$$

$$= \int \frac{e^{-x} e^{-x}}{e^{-x}+1} dx + C_2$$

putting $e^{-x} + 1 = t$

$$e^{-x} = t-1$$

$$-e^{-x} dx = dt$$

$$e^{-x} dx = -dt$$

$$= -\int \frac{(t-1)}{t} dt + C_2$$

$$= -\int \left(1 - \frac{1}{t}\right) dt + C_2$$

$$B = -[t - \log t] + C_2$$

(8) Use method of Variation of parameter find the particular Integration of $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 3x = \frac{e^t}{1+e^t}$

A.w $(D^2 - 4D + 3)x = \frac{e^t}{1+e^t}$

$M^2 - 4M - 3 = 0$

$m = 3, 1$

$\boxed{C.F = C_1 e^{3t} + C_2 e^t}$

let $u = e^{3t}$ $\quad v = e^t$

$u' = 3e^{3t}$ $\quad v' = e^t$

$W = e^{4t} - 3e^{4t}$

$W = -2e^{4t}$

$y = AU + BV$

$A = -\int \frac{RV}{W} dt + C_1$

$= -\int \frac{e^t}{(1+e^t)} \frac{e^t}{(-2e^{4t}} dt + C_1$

$= \frac{1}{2} \int \frac{1}{(1+e^t)\, e^{2t}} dt + C_1$

$= \frac{1}{2} \int \frac{e^{-2t}}{(1+e^t)} dt + C_1$

$$= \frac{1}{2}\int \frac{e^{-2t}\, e^{-t}}{e^{-t}+1}\, dt + c_1$$

putting $e^{-t}+1 = y$

$$e^{-t} = y-1$$

$$\cancel{-te}\ -e^{-t}\,dt = dy$$

$$= \frac{1}{2}\int \frac{(y-1)^2}{y}(-dy) + c_1$$

$$= -\frac{1}{2}\int \frac{(y-1)^2}{y}\, dy + c_1$$

$$= -\frac{1}{2}\int \frac{y^2+1-2y}{y}\, dy + c_1$$

$$= -\frac{1}{2}\int \left(y + \frac{1}{y} - 2\right) dy + c_1$$

$$A = -\frac{1}{2}\left[\frac{y^2}{2} + \log y - 2y\right] + c_1$$

$$B = \int \frac{Ru}{W}\, d\cancel{x} + c_2$$

$$= \int \frac{e^t}{1+e^t}\,\frac{e^{3t}}{(-2e^{4t})}\, dt + c_2$$

$$= -\frac{1}{2}\int \frac{1}{1+e^t}\, dt + c_2$$

$$= -\frac{1}{2}\int \frac{-e^{-t}}{e^{-t}+1}\, dt + c_2$$

$$= \frac{1}{2}\log(e^{-t}+1) + c_2$$

$$B = \frac{1}{2}\log(e^{-t}+1) + c_2$$

$$y = c_1e^{3t} + c_2e^{t} \cancel{\quad e^{3t}\ \frac{1}{2}}$$

$$= c_1e^{3t} + c_2e^{t} - e^{3t}\frac{1}{2}\left[\frac{y^2}{2} + \log y - 2y\right]$$
$$+ e^{t}\left[\frac{1}{2}\log(e^{-t}+1)\right.$$

$$= c_1e^{3t} + c_2e^{t} - \frac{1}{2}e^{3t}\left[\frac{(e^{-t}+1)^2}{2} + \log(e^{-t}+1) - 2(e^{-t}+1)\right]$$
$$+ e^{t}\left[\frac{1}{2}\log(e^{-t}+1)\right.$$ Ans

Q11. Solve $(D^2-1)y = 2(1-e^{-2x})^{-1/2}$ by method of variation of parameter.

$$M^2 - 1 = 0$$

$$\cancel{M = 0 \pm i} \qquad M = 1, -1$$

$$\cancel{C.F = e^{0x}[c_1\cos x + c_2\sin x]}$$

$$C.F = c_1 e^{x} + c_2 e^{-x}$$

let $u = e^{x}$ $\qquad v = e^{-x}$

$u' = e^{x}$ $\qquad v' = -e^{-x}$

$$W = -2$$

$$A = -\int \frac{Rv}{W}\,dx + c_1$$

$$= \cancel{-}\int \frac{\cancel{2}\,e^{-x}}{\sqrt{1-e^{-2x}}\,(-\cancel{2})}\,dx + c_1$$

$$= \int \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\,dx + c_1$$

puttin, $e^{-x} = t$

$$-e^{-x}\,dx = dt$$

$$e^{-x}\,dx = -dt$$

$$= -\int \frac{dt}{\sqrt{1-t^2}} + c_1$$

$$\left[\, A = -\sin^{-1} t + c_1 \,\right]$$

$$B = \int \frac{Ru}{W}\,dx + c_2$$

$$= \int \frac{2\,e^{x}}{\sqrt{1-e^{-2x}}\,(-2)}\,dx + c_2$$

$$= -\int \frac{e^{x}}{\sqrt{1-e^{-2x}}}\,dx + c_2$$

$$= -\int \frac{e^{x}\,e^{x}}{\sqrt{e^{2x}-1}}\,dx + c_2$$

$$e^{2x} - 1 = t$$

$$2e^{2x}\,dx = dt$$

$$e^{2x}\,dx = \frac{dt}{2}$$

$$= -\frac{1}{2}\int \frac{dt}{\sqrt{t}} + c_2$$

$$= -\frac{1}{2}\int t^{-1/2}\,dt + c_2$$

$$= -\frac{1}{\cancel{2}}\,\frac{t^{1/2}}{1/\cancel{2}} + c_2$$

$$B = -t^{1/2} + c_2$$

$$\left[\; y = c_1 e^{x} + c_2 e^{-x} + e^{x}\left[-\sin^{-1} e^{-x}\right] + e^{-x}\left[-e^{2x-1}\right] \;\right]$$ Ans

(9) solve by method of variation parameter for the differn equation $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 9y = \dfrac{e^{3x}}{x^2}$

A.E $(D^2 - 6D + 9)y = \dfrac{e^{3x}}{x^2}$

$m^2 - 6m + 9 = 0$

$m = 3, 3$

$C.F = c_1 e^{3x} + x c_2 e^{3x}$

let $u = e^{3x}$ $\quad v = x e^{3x}$

$u' = 3e^{3x}$ $\quad v' = 3xe^{3x} + e^{3x}$

$$W = \begin{vmatrix} e^{3x} & xe^{3x} \\ 3e^{3x} & 3xe^{3x} + e^{3x} \end{vmatrix}$$

$$= 3xe^{6x} + e^{6x} - 3xe^{6x}$$

$$\boxed{W = e^{6x}}$$

$$A = -\int \frac{RV}{W}\,dx + c_1$$

$$= -\int \frac{e^{3x}}{x^2(e^{6x})}\, x e^{3x}\,dx + c_1$$

$$= -\int \frac{1}{x}\,dx + c_1$$

$$= -\log(x) + c_1$$

$$B = \int \frac{Ru}{\omega} dx + c_2$$

$$= \int \frac{e^{3x}}{x^2 e^{6x}} e^{3x} dx + c_2$$

$$= \int \frac{1}{x^2} dx + c_2$$

$$= \int x^{-2} dx + c_2$$

$$= \frac{x^{-1}}{-2+1} + c_2$$

$$B = -\frac{1}{x} + c_2$$

$$y = c_1 e^{3x} + x c_2 e^{3x} + -e^{3x} \log(x) - x e^{3x} \frac{1}{x}$$

$$\left[y = c_1 e^{3x} + x c_2 e^{3x} - e^{3x} \log(x) - e^{3x} \right]$$ Ans

Q 10 Solve the diffn eqn $(D^2+2D+2)y = e^{-x} \sec^3 x$ where $D = \frac{d}{dx}$

Sol $(D^2+2D+2)y = e^{-x} \sec^3 x$

$m^2 + 2m + 2 = 0$

$m = -1 \pm i$

$C.F = e^{-x} [c_1 \cos x + c_2 \sin x]$

$u = \cos x \qquad v = \sin x$

$u' = -\sin x \qquad v' = \cos x$

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$W = \cos^2 x + \sin^2 x$

$\left[W = 1 \right]$

$$A = -\int \frac{RV}{W}\,dx + c_1$$

$$= -\int e^{-x} \cancel{\sec^3 x\ \sin x\ dx} + c_1$$

$$= -\int e^{-x} \frac{\cancel{\sin x\ \sec^2 x\ dx + c_1}}{\cos x}$$

$$= -\int e^{-x} \cancel{\tan x\ \sec^2 x\ dx} + c_1$$

$$C.F = e^{-x}\,(c_1 \cos x + c_2 \sin x]$$

$u = e^{-x} \cos x$ $\qquad$ $v = e^{-x} \sin x$

$u' = e^{-x}(-\sin x) + e^{-x} \cos x$ $\qquad$ $v' = e^{-x} \cos x + e^{-x} \sin x$

$= -e^{-x}[\cos x - \sin x]$ $\qquad$ $= e^{-x}[\cos x - \sin x]$

$$W = \begin{vmatrix} e^{-x} \cos x & e^{-x} \sin x \\ -e^{-x} \sin x + e^{-x} \cos x & -e^{-x} \sin x + e^{-x} \cos x \end{vmatrix}$$

$$[\, w = e^{-2x} \,]$$

$$A = -\int \frac{RV}{W}\,dx + c_1$$

$$= -\int \frac{e^{-x}\ \cancel{e^{-x}}\ \sec x}{e^{-2x}}\,dx + c_1$$

$$= -\int \cancel{\sin x\,dx + c_1} \qquad = -\int \sec^3 x\,dx + c_1$$

$$= -\int \cancel{-(\cos x) + c_1}$$

$$A = \cancel{\cos x + c_1}$$

Let $y = Au + Bv$

$$A = -\int \frac{Rv}{w}\,dx + c_1$$

$$= -\int \frac{e^{-x} \cdot xe^{3x}\, e^{-x} \sin 2x\, dx}{e^{-2x}}$$

$$= -\int \sec^3 x \cdot \sin x\, dx$$

$$A = -\int \tan x \cdot \sec^2 x\, dx$$

let $\tan x = t$

$\sec^2 x\, dx = dt$

$$A = \int t\, dt$$

$$A = -\frac{t^2}{2} + c_1$$

$$A = -\frac{\tan^2 x}{2} + c_1$$

$$B = \int \frac{Ru}{w}\,dx + c_2$$

$$= \int \frac{e^{-x} \cdot \sec^3 x \cdot e^{-x} \cos x}{e^{-2x}}$$

$$= \int \sec^2 x\, dx$$

$$= \tan x\, dx + c_2$$

(3) obtain the genral soln of the differential equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^3 e^x$$

(13) Solve by method of variation of parameter for the diff^n equation $x^2y'' + xy' - y = x^2 e^x$

(14) Apply method of variation of Parameter to solve

$$x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$$

(15) using method of variation of Parameter sol

$$x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 12y = 0$$

Solve (3) $\quad x^2y'' + xy' - y = x^3e^x$

$$R = \frac{x^3e^x}{x^2} = xe^x$$

let $\quad x^2y'' + xy' - y = 0$

putting $\quad x^2y'' + xy' - y = 0$

putting. $x = e^z \;,\; z = \log x$

$$x\frac{dy}{dx} = D$$

$$x^2\frac{d^2y}{dx^2} = D(D-1)$$

$$[D^2 - \cancel{D} + \cancel{D} - 1)y = 0$$

$$(D^2-1)y = 0$$

A.E $\quad m^2 - 1 = 0$

$$m = 1, -1$$

$$C.F = c_1e^z + c_2e^{-z}$$

$$= c_1x + \frac{c_2}{x}$$

Let $\quad u = x \qquad v = 1/x$

$\quad u' = 1 \qquad v' = -1/x^2$

$$W = -\frac{1}{x} - \frac{1}{x} \Rightarrow \frac{-2}{x}$$

$$y = AU + BV$$

$$A = -\int \frac{RV}{W}\,dx + C_1$$

$$= -\int \frac{xe^x \, 1/x}{-2/x}\,dx + C_1$$

$$A = \frac{1}{2}\int xe^x\,dx + C_1$$

$$A = \cancel{xe^x} + \frac{1}{2}(xe^x - e^x) + C_1$$

$$B = \int \frac{RU}{W}\,dx + C_2$$

$$= \int \frac{xe^x x}{-2/x}\,dx + C_2$$

$$= -\frac{1}{2}\int x^3 e^x\,dx + C_2$$

$$B = -\frac{1}{2}\left[x^3e^x - 3x^2e^x + 6xe^x - 6e^x\right] + C_2$$

$$\left[\, y = C_1 x + \frac{C_2}{x} + \frac{1}{2}\left[x^2e^x - xe^x\right] - \frac{1}{2}\left[x^2e^x - 3xe^x + 6e^x - 6xe^x\right]\right]$$

Ans

Sol^n:-

$$x^2y'' + xy' - y = x^2e^x$$

$$R = \frac{x^2e^x}{x^2} = e^x$$

$$x^2y'' + xy' - y = 0 \qquad \text{putting } x = e^z \quad z = \log x$$

$$x\frac{d}{dx} = D$$

$$x^2\frac{d}{dx} = D(D-1) = D^2 - D$$

$$(D^2 - D + D - 1)y = 0$$

$$A.E \quad m^2 - 1 = 0$$

$$m = 1, -1$$

$$C.F = c_1 e^z + c_2 e^{-z}$$

$$C.F = c_1 x + \frac{c_2}{x}$$

let $u = x$ $\qquad$ $v = 1/x$

$u' = 1$ $\qquad$ $v' = -1/x^2$

$$W = -\frac{1}{x} - \frac{1}{x}$$

$$\left[W = \frac{-2}{x} \right]$$

$$y = Au + Bv$$

$$A = -\int \frac{Rv}{W} dx + c_1$$

$$= -\int \frac{e^x \, 1/x}{-2/x} dx + c_1$$

$$= \frac{1}{2} \int e^x dx + c_1$$

$$A = \frac{1}{2} e^x + c_1$$

$$B = \int \frac{Ru}{W} dx + c_2$$

$$= \int \frac{e^x x}{-2/x} dx + c_2$$

$$= -\frac{1}{2} \int x^2 e^x dx + c_2$$

$$B = -\frac{1}{2} \left[x^2 e^x - 2x e^x + 2e^x \right] + c_2$$

$$\left[y = \frac{x}{2}e^{x} + c_1 x - \frac{1}{2}\left[x e^{x} - 2e^{x} + \frac{2}{x}e^{x} \right] + \frac{c_2}{x} \right]$$ **Ans**

Solve 14 $\quad x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^{x}$

$$R = \frac{e^{x}}{x^2}$$

let $\quad x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 0$

putting $x = e^{z}$ $\quad z = \log x$

$$x\frac{d}{dx} = D$$

$$x^2\frac{d}{dx} = (D^2 - D)$$

~~$D^2 + 4$~~ $\quad (D^2 - D + 4D + 2)y = 0$

~~A.E~~ $\quad (D^2 + 3D + 2)y = 0$

A.E $\quad M^2 + 3D + 2 = 0$

$$M = -1, -2$$

$$C.F = c_1 e^{-z} + c_2 e^{-2z}$$

$$C.F = \frac{c_1}{x} + \frac{c_2}{x^2}$$

let $\quad u = 1/x \qquad v = 1/x^2$

$$u' = -\frac{1}{x^2} \qquad v' = \frac{-2}{x^3}$$

$$W = \begin{vmatrix} \frac{1}{x} & 1/x^2 \\ -1/x^2 & -2/x^3 \end{vmatrix}$$

$$W = \frac{-2}{x^4} + \frac{1}{x^4}$$

$$\left[W = -\frac{1}{x^4} \right]$$

$$A = -\int \frac{RV}{W}\,dx + c_1$$

$$= -\int \frac{\frac{e^x}{x^2} \times \frac{1}{x^2}}{-1/x^4}\,dx + c_1$$

$$A = \int e^x\,dx + c_1$$

$$\left[\; A = e^x + c_1 \;\right]$$

$$B = \int \frac{RU}{W}\,dx + c_2$$

$$= \int \frac{\frac{e^x}{x^2}\,\frac{1}{x}}{-1/x^4}\,dx + c_2$$

$$= -\int x e^x\,dx + c_2$$

$$= -\left[x e^x - 1 e^x\right] + c_2$$

$$B = -x e^x + e^x + c_2$$

$$y = \frac{c_1}{x} + \frac{c_2}{x^2} + \cancel{\frac{e^x}{x}} + \frac{e^x}{x^2} - \cancel{\frac{e^x}{x}}.$$

$$\left[\; y = \frac{c_1}{x} + \frac{c_2}{x^2} + \frac{e^x}{x^2} \;\right]$$

(1.5) solve $\quad x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 12y = 0$

$$[\,R = 0\,]$$

$$x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 12y = 0$$

[putting $x = e^z \cdot z = \log x$]

$x\frac{d}{dx} = D$

$x^2\frac{d}{dx^2} = D^2 - D$

$(D^2 - D + 2D - 12)y = 0$

$(D^2 + D - 12)y = 0$

A.E $\quad M^2 + M + 12 = 0$

$M = 3, -4$

$C.F = c_1e^{3z} + c_2e^{-4z}$

$C.F = c_1x^3 + \frac{c_2}{x^4}$

$u = x^3 \quad v = 1/x^4$

$u' = 3x^2 \quad v' = -4/x^5$

$$W = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix}$$

$$= \frac{-4}{x^2} - \frac{3}{x^2} = \frac{-7}{x^2}$$

$y = Au + Bv$

$A = -\int \frac{RV}{W} dx + c_1$

$y = Au + Bv$

$y = c_1 + c_2$

$$\left[y = c_1x^3 + \frac{c_2}{x^4} \right]$$

Q12.

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x$$

A.E $\quad m^2 - 2m = 0$

$m(m-2) = 0$

$m = 0, 2$

$C.F = c_1 + c_2e^{2x}$

$u = 1 \quad v = e^{2x}$

$u' = 0 \quad v' = 2e^{2x}$

$$\left[W = 2e^{2x} \right]$$

$y = Au + Bv$

$A = -\int \frac{RV}{W} dx + c_1$

$= -\int \frac{e^x \sin x \, e^{2x}}{2e^{2x}} dx + c_1$

$= -\frac{1}{2}\int e^x \sin x \, dx + c$

$$= -\frac{1}{2}\frac{e^{x}}{1+1}\left(\sin x-\cos x\right)+c_1$$

$$A = \frac{-e^{x}}{4}\left[\sin x-\cos x\right]+c_1$$

$$B = \int \frac{RU}{W}\,dx + c_2$$

$$= \int \frac{e^{x}\sin x}{2e^{2x}}\,dx + c_2$$

$$= \frac{1}{2}\int e^{-x}\sin x\,dx + c_2$$

$$= \frac{1}{2}\left[\frac{e^{-x}}{1+1}\left(-\sin x-\cos x\right)\right]+c_2$$

$$= \frac{-e^{-x}}{4}\left[\sin x+\cos x\right]+c_2$$

$$\left[\, y = c_1 + c_2 e^{2x} - \frac{e^{x}}{4}\left[\sin x-\cos x\right] - \frac{e^{-x}}{4}\left[\sin x+\cos x\right] \right]$$

H.W $\quad x^2y'' + xy' - 9y = 48x^5$

$(D^2+1)y = \tan^2 x$

(By changing The Independent Variables)

step for soln:

(I) make the coficieant of $\frac{d^2y}{dx^2}$ as 1.

(II) Compare with standard form

$$\left[\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R\right]$$

and find P, Q R

(III) Write $\left(\frac{dz}{dx}\right)^2 = Q$ Here Q = Holesquare of a function without radical sign and negative sign.

(IV) find $\frac{dz}{dx}$ $\xrightarrow{\text{Integrate}}$ Hence obtain z by integration

and $\frac{d^2y}{dx^2}$ by diffn.

(V) find ~~Q_1, R_1~~ P_1, Q_1. & R_1

$$P_1 = \frac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2}$$

$$Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2} \qquad R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2}$$

VI) Reduce diffn equation $\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1 y = R_1$

VII) find C.F and PI in term of z.

Q① Solve by changing the independent variable $\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} + 4x^2y = x^4$

Sol^n: $\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} + 4x^2y = x^4$

Standard form $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$

Compare $P = \frac{-1}{x}$ $\quad Q = 4x^2 \quad R = x^4$

$\left(\frac{dz}{dx}\right)^2 = 4x^2$

$\frac{dz}{dx} = 2x$ $\xrightarrow{\text{Integration}} \boxed{z = x^2}$

$\frac{dz}{dx} = 2x$ $\xrightarrow{\text{Diff}} \boxed{\frac{d^2z}{dx^2} = 2}$

$$P_1 = \frac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2} = \frac{2 - \frac{1}{x} \times 2x}{4x^2} = 0$$

$[P_1 = 0]$

$$Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2} = \frac{4x^2}{4x^2} = 1 \Rightarrow [Q_1 = 1]$$

$$R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2} = \frac{x^4}{4x^2} = \frac{1}{4}x^2 \quad \left[R_1 = \frac{1}{4}z\right] \text{ since } x^2 = z$$

Reduced diff equn

$$\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1 y = R_1$$

$$\frac{d^2y}{dz^2} + y = \frac{1}{4}z$$

A.E $\quad m^2+1=0$

$m = \pm i$

$$\left[C.F = c_1\cos z + c_2\sin z \right]$$

$$P.I = \frac{1}{D^2+1}\frac{1}{4}z$$

$$= \frac{1}{4}\frac{1}{D^2+1}z$$

$$= \frac{1}{4}\left[1+D^2\right]^{-1}z$$

$$\left[P.I = \frac{1}{4}z \right]$$

$$y = c_1\cos z + c_2\sin z + \frac{1}{4}z$$

$$\left[y = c_1\cos x^2 + c_2\sin x^2 + \frac{1}{4}x^2 \right] \text{ Ans}$$

(2) Solve by changing the independent variable

$$\cos x\frac{d^2y}{dx^2} + \sin x\frac{dy}{dx} + 2y\cos^3 x = 2\cos^5 x$$

sol

$$\frac{d^2y}{dx^2} + \tan x\frac{dy}{dx} + 2y\cos^2 x = 2\cos^4 x$$

standard form

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R \quad \text{compare}$$

$$P = \frac{\sin x}{\cos x} \qquad Q = 2\cos^2 x \qquad R = 2\cos^4 x$$

$$\left(\frac{dz}{dx}\right)^2 = \cos^2 x$$

$$\frac{dz}{dx} = \cos x \quad \xrightarrow{\text{Integration}} \quad z = \sin x$$

$$\frac{dz}{dx} = \cos x \quad \xrightarrow{\text{diff}} \quad \frac{d^2z}{dx^2} = -\sin x$$

$$P_1 = \frac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2} = \frac{-\sin x + \frac{\sin x}{\cos x} \times \cos x}{\cos^2 x}$$

$$\boxed{P_1 = 0}$$

$$Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2} = \frac{2\cos^2 x}{\cos^2 x} = 2$$

$$\boxed{Q_1 = 2}$$

$$R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2} = \frac{2\cos^4 x}{\cos^2 x} = 2\cos^2 x$$

$$R_1 = 2\cos^2 x$$

$$R_1 = 2(1-\sin^2 x)$$

$$R_1 = 2(1-z^2)$$

$$\boxed{\cancel{R_1 = 2 - 2z^2}}$$

Reduced diff eqn

$$\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1 y = R_1$$

$$\frac{d^2y}{dz^2} + 2y = 2(1-z^2)$$

$$\cancel{M^2 + 2y} \cdot \quad M^2 + 2 = 0$$

$$M = 0 \pm i\sqrt{2}$$

$$\boxed{C.F = C_1\cos\sqrt{2}z + C_2\sin\sqrt{2}z}$$

$$P.I = 2\frac{1}{D^2+2}(1-z^2)$$

$$= \frac{2}{2\left[1+\frac{D^2}{2}\right]}(1-z^2)$$

$$= \left[1+\frac{D^2}{2}\right]^{-1}(1-z^2)$$

$$= \left[1-\frac{D^2}{2}\right](1-z^2)$$

$$= 1-z^2+1$$

$$\left[P.I = 2-z^2\right]$$

$$y = C_1\cos\sqrt{2}z + C_2\sin\sqrt{2}z + 2 - z^2$$

$$\left[y = C_1\cos\sqrt{2}\sin x + C_2\sin\sqrt{2}\sin x + 2 - \sin^2 x\right]$$

Ans

Q3. Solve by changing the independent variable

$$x\frac{d^2y}{dx^2} + (4x^2-1)\frac{dy}{dx} + 4x^3y = 2x^3$$

$$= \frac{d^2y}{dx^2} + \left(4x-\frac{1}{x}\right)\frac{dy}{dx} + 4x^2y = 2x^2$$

Standard equation $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$

compare

$$P = \left(4x-\frac{1}{x}\right) \qquad Q = 4x^2 \qquad R = 2x^2$$

$$\left(\frac{dz}{dx}\right)^2 = 4x^2$$

$$\frac{dz}{dx} = 2x \xrightarrow{\text{Integration}} z = x^2$$

$$\frac{dz}{dx} = 2x \xrightarrow{\text{diff}} \left[\frac{d^2z}{dx^2} = 2\right]$$

$$P_1 = \frac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2} = \frac{2 + \left(4x - \frac{1}{x}\right) 2x}{4x^2}$$

$$= \frac{2 + \left(\frac{4x^2 - 1}{x}\right) \times 2x}{4x^2}$$

$$= \frac{\cancel{2} + 8x^2 \cancel{- 2}}{4x^2}$$

$$\left[P_1 = 2 \right]$$

$$Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2} = \frac{4x^2}{4x^2} = 1$$

$$R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2} = \frac{2x^2}{4x^2} = \frac{1}{2}$$

Reduced diff eqn

$$\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1 y = R_1$$

$$\frac{d^2y}{dz^2} + 2\frac{dy}{dz} + y = \frac{1}{2}$$

$$(D^2 + 2D + 1)y = 0$$

A.E $\quad M^2 + 2M + 1 = 0$

$$M = -1, -1$$

$$C.F = C_1 e^{-z} + z C_2 e^{-z}$$

$$P.I = \frac{1}{D^2 + 2D + 1} \frac{1}{2} e^{0z}$$

$\cancel{D^2 = a^2 = 0}$ $\quad D = a = 0$

$$= \cancel{\frac{1}{2} \frac{1}{2D+1}} \left[P.I = \frac{1}{2} \right]$$

$$P.I = \frac{1}{2}$$

$$y = C.F + P.I$$
$$y = c_1 e^{-z} + z c_2 e^{-z} + \frac{1}{2}$$
$$\left[y = c_1 e^{-x^2} + x^2 c_2 e^{-x^2} + 1/2 \right] \quad \underline{\underline{\text{Ans}}}$$

(4) Solve by changing indepent variable

$$\frac{d^2y}{dx^2} + (3\sin x - \cot x)\frac{dy}{dx} + 2y\sin^2 x = e^{-\cos x}\sin^2 x$$

<u>Solve</u> Standard form $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$

Compare and get

$$P = (3\sin x - \cot x) \quad Q = 2\sin^2 x \quad R = e^{-\cos x}\sin^2 x$$

$$\cancel{P_1 = \frac{d^2z}{dx^2} + P\frac{dz}{dx}}$$

$$\left(\frac{dz}{dx}\right)^2 = \sin^2 x$$

integration $z = -\cos x$

$$\frac{dz}{dx} = \sin x$$

diff $\frac{d^2z}{dx^2} = \cos x$

$$P_1 = \frac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2}$$

$$= \frac{\cos x + (3\sin x - \cot x)\sin x}{\sin^2 x}$$

$$= \frac{\cos x + \left(3\sin x - \frac{\cos x}{\sin x}\right)\sin x}{\sin^2 x}$$

$$= \frac{\cos x + \left[\frac{3\sin^2 x - \cos x]\sin x}{\sin x}\right.}{\sin^2 x}$$

$$= \frac{\cos x + 3\sin^2 x - \cos x}{\sin^2 x}$$

$$\left[P_1 = 3\right]$$

$$Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2} = \frac{2\sin^2 x}{\sin^2 x} = 2$$

$$\left[Q_1 = 2\right]$$

$$R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2} = \frac{e^{-\cos x}\sin^2 x}{\sin^2 x}$$

$$\left[\cancel{R_1} = e^{-\cos x}\right] = e^z$$

Reduce eq diff eqn

$$\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1 y = R_1$$

$$\frac{d^2y}{dz^2} + 3\frac{dy}{dz} + 2y = \cancel{e^{-\cos x}}\, e^z$$

$$(D^2 + 3D + 2)y = 0$$

$$m^2 + 3m + 2 = 0$$

$$m = -1, -2$$

$$C.F = c_1 e^{-z} + c_2 e^{-2z}$$

$$P.I = \frac{1}{D^2 + 3D + 2} e^{\cancel{-\cos x}}\, e^z$$

$$D = a = 1$$

$$P.I = \frac{1}{1+3+2} e^{z}$$

$$P.I = \frac{1}{6} e^{z}$$

$$y = C.F + P.I$$

$$= c_1 e^{-z} + c_2 e^{-2z} + \frac{1}{6} e^{z}$$

$$\left[y = c_1 e^{\cos x} + c_2 e^{2\cos x} + \frac{1}{6} e^{\cos x} \right]$$

(6)

Solve $\frac{d^2y}{dx^2}$ -

Solve by changing the independent variable

$$x^6 \frac{d^2y}{dx^2} + 3x^5 \frac{dy}{dx} + a^2 y = \frac{1}{x^2}$$

Sol^n. $$\frac{d^2y}{dx^2} + \frac{3}{x} \frac{dy}{dx} + \frac{a^2}{x^6} y = \frac{1}{x^8}$$

Standard form $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$ Compare

we get

$$P = \frac{3}{x} \qquad Q = \frac{a^2}{x^6} \qquad R = \frac{1}{x^8}$$

$$\left(\frac{dz}{dx}\right)^2 = \frac{a^2}{x} \qquad z = ax$$

Inten

$$\frac{dz}{dx} = a$$

$$\frac{d^2z}{dx^2} = 0$$

$$\left(\frac{dz}{dx}\right)^2 = \frac{a^2}{x^6}$$

$$z = \frac{a}{-2x^2}$$

integrati

$$\frac{dz}{dx} = \frac{a}{x^3}$$

$$\frac{d^2z}{dx^2} = \frac{-3a}{x^4}$$

$$P_1 = \frac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2} = \frac{\frac{-3a}{x^4} + \frac{3}{x}\frac{a}{x^3}}{a/x^3} = 0$$

$$[P_1 = 0]$$

$$Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2} = 1$$

$$R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2} = \frac{1/x^8}{a^2/x^6}$$

$$R_1 = \frac{1}{x^2 a^2}$$

$$R_1 = \frac{1}{\frac{a}{-2z} a^2}$$

$$\left[R_1 = -\frac{2z}{a^3}\right]$$

Reduce diff equ

$$\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Qy = R_1$$

$$\frac{d^2y}{dz^2} + y = -\frac{2z}{a^3}$$

$$(D^2+1)y = 0$$

D^2

A.E $\quad M = 0 \pm i$

$$[C.F = c_1\cos z + c_2\sin z]$$

$$P.I = \frac{1}{D^2+1}\left(\frac{-2}{a^3}\right)z$$

$$= \frac{-2}{a^3}\,\frac{1}{D^2+1}\,z$$

$$= \frac{-2}{a^3}\,[1+D^2]^{-1}z$$

$$= \frac{-2z}{a^3}$$

$$P.I = \frac{-2z}{a^3}$$

$$y = C.F + P.I$$

$$= c_1\cos z + c_2\sin z - \frac{2z}{a^3}$$

$$= c_1\cos\left(\frac{a}{-2x^2}\right) + c_2\sin\left(-\frac{a}{2x^2}\right) - \frac{2\times\left(-\frac{a}{2x^2}\right)}{a^3}$$

$$\boxed{y = c_1\cos\left(\frac{-a}{2x^2}\right) + c_2\left(\sin\frac{-a}{2x^2}\right) + \frac{1}{x^2a^2}}$$

Ans

7 Solve the following diffⁿ equation by changing the independent variable:

$$x\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3y = 8x^3\sin x^2$$

Solⁿ
$$\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} - 4x^2y = 8x^2\sin x^2$$

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

$$P = -\frac{1}{x} \qquad Q = -4x^2 \qquad R = 8x^2\sin x^2$$

let $\left(\frac{dz}{dx}\right)^2 = 4x^2$

$$\frac{dz}{dx} = 2x \qquad \text{inte} \quad z = x^2$$

$$\frac{d^2z}{dx^2} = 2$$

$$P_1 = \frac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2} = \frac{2 - \frac{1}{2}2x}{4x^2} = 0$$

$[P_1 = 0]$

$$Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2} = \frac{-4x^2}{4x^2} = -1$$

$[Q_1 = -1]$

$$R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2} = \frac{8x^2 \sin x^2}{4x^2}$$

$[R_1 = 2\sin x^2] = 2\sin z]$

Reduced diff eqn.

$$\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1 y = R_1$$

$$\frac{d^2y}{dx^2} - y = 2\sin z$$

A.E $\quad m^2 - 1 = 0$

$m = 1, -1$

C.F $= c_1 e^z + c_2 e^{-z}$

$$P.I = \frac{1}{D^2 - 1} 2\sin z$$

$D^2 = -a^2 = -1$

$$= -\frac{1}{2} 2\sin z$$

$[P_1 = -\sin z]$

$$\left[y = c_1 e^z + c_2 e^{-z} - \sin z\right]$$

(5) $\frac{d^2y}{dx^2} - \cot x \frac{dy}{dx} - y\sin^2 x = \cos x \cdot \cos^3 x$

Sol^n: $\frac{d^2y}{dx^2} - \cot x \frac{dy}{dx} - y\sin^2 x = \cos x [1 - \cos^2 x]$

$$\frac{d^2y}{dx^2} - \cot x \frac{dy}{dx} - y\sin^2 x = \cos x \sin^2 x$$

Standard form-

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$ compare and get

$P = -\cot x \qquad Q = -\sin^2 x \qquad R = \cos x \sin^2 x$

$$\left(\frac{dz}{dx}\right)^2 = \sin^2 x$$

$$\frac{dz}{dx} = \sin x \xrightarrow{\text{Inte}} z = -\cos x$$

diff $\frac{d^2z}{dx^2} = \cos x$

$$P_1 = \frac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2} = \frac{\cos x + (-\cot x)\cos x}{\sin^2 x}$$

$$= \frac{\cos x - \frac{\cos x \cos x}{\sin x}}{\sin^2 x}$$

$$= \frac{\cos x - \frac{\cos^2 x}{\sin x}}{\sin^2 x}$$

$$= \cancel{\frac{\sin x \cos x - \cos^2 x}{\sin^3 x}}$$

$= \cos x [$

$[P_1 = 0]$

$$Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2} = \frac{-\sin^2 x}{\sin^2 x} = -1$$

$$R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2} = \frac{\cos x \sin^2 x}{\sin^2 x}$$

$$R_1 = \cos x$$

$$\left[R_1 = -z \right]$$

Reduce the diff. equat.

$$\frac{d^2y}{dz^2} + P_1 \frac{dy}{dz} + Q_1 y = R_1$$

$$\frac{d^2y}{dz^2} - y = -z$$

$$(D^2 - 1) y = -z$$

A.E $\quad M^2 - 1 = 0$

$$M = 1, -1$$

$$\left[C.F = c_1 e^{z} + c_2 e^{-z} \right]$$

$$P.I = \frac{1}{D^2 - 1} - z$$

$$= \frac{1}{-1[1 - D^2]} [-z]$$

$$= [1 - D^2]^{-1} z$$

$$= [1 + D^2] z$$

$$\left[P.I = z \right]$$

$$y = c_1 e^{z} + c_2 e^{-z} + z$$

$$\left[y = c_1 e^{-\cos x} + c_2 e^{\cos x} - \cos x \right]$$ Ans

(Simultaneous differential equation)

Q1 solve $\frac{dx}{dt} = -4(x+y)$; $\frac{dx}{dt} + 4\frac{dy}{dt} = -4y$ with conditions

$x(0) = 1$, $y(0) = 0$

Soln

$$\frac{dx}{dt} = -4(x+y)$$

$$\frac{dx}{dt} + 4\frac{dy}{dt} = -4y$$

$$Dx + 4x + 4y = 0 \quad \text{—— (1)}$$

$$Dx + 4Dy + 4y = 0 \quad \text{—— (2)}$$

$$(D+4)x + 4y = 0 \qquad \times (D+1)$$

$$Dx + 4(D+1)y = 0 \qquad \times 1$$

$$(D^2+5D+4)x + 4(D+1)y = 0$$

$$\underline{\overset{-}{}Dx \quad \overset{-}{+} \quad 4(D+1)y = 0}$$

$$(D^2+4D+4)x = 0$$

A.E. $\quad m^2 + 4m + 4 = 0$

$m = -2, -2$

$C.F = C_1 e^{-2t} + t C_2 e^{-2t}$

$P.I = 0$

$x = C.F + P.I$

$$[\ x = C_1 e^{-2t} + t C_2 e^{-2t}\ ] \quad \text{—— (11)}$$

$$\frac{dx}{dt} = -2C_1 e^{-2t} - 2t C_2 e^{-2t} + C_2 e^{-2t}$$

from equ (1)

$$-2c_1e^{-2t} - 2tc_2e^{-2t} + c_2e^{-2t}$$

$$4c_1e^{-2t} + 4tc_2e^{-2t} + 4y = 0$$

$$2c_1e^{-2t} + 2tc_2e^{-2t} + c_2e^{-2t} + 4y = 0$$

$$4y = -2c_1e^{-2t} - 2tc_2e^{-2t} - c_2e^{-2t}$$

$$y = \frac{-c_1e^{-2t}}{2} - \frac{tc_2e^{-2t}}{2} - \frac{c_2}{4}e^{-2t} \quad \text{—— (2)}$$

(1) $x(0) = 1$

$[1 = c_1]$

(2) $y(0) = 0$

$$0 = -\frac{c_1}{2} - \frac{c_2}{4}$$

$$0 = -\frac{1}{2} - \frac{c_2}{4} \Rightarrow \frac{c_2}{4} = -\frac{1}{2}$$

$[c_2 = -2]$

$$\left[x = e^{-2t} - 2te^{-2t}\right]$$

$$\left[y = -\frac{e^{2t}}{2} + te^{-2t} + \frac{1}{2}e^{-2t}\right]$$

2) solve $\frac{dx}{dt} - y = e^t$ and $\frac{dy}{dt} + x = \sin t$ given $x(0) = 1$ $y(0) = 0$

Sol -
$$\frac{dx}{dt} - y = e^t \quad \text{—— (1)}$$

$$\frac{dy}{dt} + x = \sin t \quad \text{—— (11)}$$

$$Dx - y = e^t \quad \times D \quad —①$$

$$Dx + x = \sin t \quad \times 1 \quad —②$$

$$D^2x - Dy = De^t$$

$$Dy + x = \sin t$$

$$(D^2+1)x = e^t + \sin t$$

A.E $\quad m^2 + 1 = 0$

$$m = 0 \pm i$$

$$\left[\ C.F = (c_1 \cos t + c_2 \sin t)\ \right]$$

$$P.I = \frac{1}{D^2+1} e^t + \frac{1}{D^2+1} \sin t$$

$$D = a = 1 \qquad D^2 = -a^2 = -1$$

$$= \frac{1}{1+1} e^t + \frac{1}{-1+1} \sin t \quad \text{case fail}$$

$$= \frac{e^t}{2} + \frac{t}{2D} \sin t$$

$$= \frac{e^t}{2} + \frac{t}{2}(-\cos t)$$

$$\left[\ P.I = \frac{e^t}{2} - \frac{t\cos t}{2}\ \right]$$

$$\cancel{y = C.F + P.I}$$

$$\cancel{y = c_1 \cos t + c_2 \sin t + e^t}$$

$$x = c_1 \cos t + c_2 \sin t + \frac{e^t}{2} - \frac{t \cos t}{2} \quad —①$$

$$\frac{dx}{dt} = c_1(-\sin t) + c_2 \cos t + \frac{e^t}{2} - \frac{1}{2}\left[t(-\sin t) + \cos t\right]$$

$$\frac{dx}{dt} = -c_1 \sin t + c_2 \cos t + \frac{e^t}{2} + \frac{1}{2} t \sin t - \frac{1}{2} \cos t$$

from equation (1)

$$\frac{dx}{dt} - y = e^t$$

$$y = \frac{dx}{dt} - e^t$$

$$y = -c_1 \sin t + c_2 \cos t + \frac{e^t}{2} + \frac{1}{2} t \sin t - \frac{1}{2} \cos t - e^t \quad \text{--- (2)}$$

$$\left[y = -c_1 \sin t + c_2 \cos t - \frac{e^t}{2} + \frac{1}{2} t \sin t - \frac{\cos t}{2} \right]$$

(i) $x(0) = 1$

$$1 = c_1 + \frac{1}{2}$$

$$\cancel{c_1 = -1/} \quad [c_1 = 1/2]$$

(ii)

$$y(0) = 0$$

$$0 = -\frac{1}{2} - \frac{1}{2} + c_2$$

$$[c_2 = 1]$$

$$\left[y = -\frac{1}{2} \sin t + \cos t - \frac{e^t}{2} + \frac{1}{2} t \sin t - \frac{\cos t}{2} \right]$$

$$\left[x = -\frac{\sin t}{2} + \cos t + \frac{e^t}{2} + \frac{t \sin t}{2} - \frac{\cos t}{2} \right]$$

(3) solve simultaneous differential eqn

$$\frac{dx}{dt} + 2x - 3y = t \quad \text{--- (1)} \qquad \frac{dy}{dt} - 3x + 2y = e^{2t}$$

$x = y = 0$ when $t = 0$

Sol $Dx + 2x - 3y = t$

$Dy - 3x + 2y = e^{2t}$

~~$(D+2)y - 3y = t$~~

$(D+2)x - 3y = t \quad \times (D+2)$

$-3x + (D+2)y = e^{2t} \quad \times 3$

$(D+2)^2 x - (D+2)3y = (D+2)t$

$-9x + (D+2)3y = 3e^{2t}$

$[(D+2)^2 - 9]x = (D+2)t + 3e^{2t}$

$[(D+2)^2 - 9]x = 1 + 2t + 3e^{2t}$

$[D^2 + 4 + 4D - 9]x = (1+2t) + 3e^{2t}$

$[D^2 + 4D - 5]x = (1+2t) + 3e^{2t}$

A.E $\quad m^2 + 4m - 5 = 0$

$m = 1, -5$

$C.F = c_1 e^{t} + c_2 e^{-5t}$

$$P.I = \frac{1}{D^2+4D-5}(1+2t) + \frac{1}{D^2+4D-5} 3e^{2t}$$

$$P.I = \frac{1}{-5\left[1 + \frac{D^2+4D}{-5}\right]}(1+2t) + 3\frac{1}{4+8-5}e^{2t}$$

$$P.I = \frac{1}{-5}\left[1 + \frac{D^2+4D}{-5}\right]^{-1}[1+2t] + \frac{3}{7}e^{2t}$$

$$= -\frac{1}{5}\left[1 + \frac{D^2+4D}{5}\right](1+2t) + \frac{3}{7}e^{2t}$$

$$= -\frac{1}{5}\left[1 + \frac{4D}{5}\right](1+2t) + \frac{3}{7}e^{2t}$$

$$P.I = -\frac{1}{5}\left[1+2t+\frac{8}{5}\right]+\frac{3}{7}e^{2t} \Rightarrow -\frac{1}{5}\left[2t+\frac{13}{5}\right]+\frac{3}{7}e^{2t}$$

~~$x = C.F + P.I$~~

$$= -\frac{2t}{5} - \frac{13}{25} + \frac{3}{7}e^{2t}$$

$$x = C.F + P.I$$

$$x = c_1e^{t} + c_2e^{-5t} - \frac{2t}{5} + \frac{3}{7}e^{2t} - \frac{13}{25}$$

$$\frac{dx}{dt} = c_1e^{t} - 5c_2e^{-5t} + \frac{6}{7}e^{2t} - \frac{2}{5}$$

from equation (1)

$$\frac{dx}{dt} + 2x - 3y = t$$

$$y = \frac{1}{3}\frac{dx}{dt} + \frac{2}{3}x - \frac{t}{3}$$

$$y = \frac{1}{3}\left[c_1e^{t} - 5c_2e^{-5t} + \frac{6}{7}e^{2t} - \frac{2}{5}\right] + \frac{2}{3}\left[c_1e^{t} + c_2e^{-5t} - \frac{2t}{5} + \frac{3}{7}e^{2t} - \frac{13}{25}\right] - \frac{t}{3}$$

$$y = c_1e^{t} - c_2e^{-5t} - \frac{12}{25} + \frac{4}{7}e^{2t} - \frac{3t}{5}$$

$$y = c_1e^{t} - c_2e^{-5t} + \frac{4}{7}e^{2t} - \frac{3t}{5} - \frac{12}{25}$$

(I) $x(0) = 0$

$$0 = c_1 + c_2 - \frac{16}{175}$$

$$\left[c_1 + c_2 = \frac{16}{175}\right]$$

(II) $y(0) = 0$

$$0 = c_1 - c_2 + \frac{168}{175}$$

$$c_1 - c_2 = -\frac{16}{175}$$

$$\left[c_2 = \cancel{175} \frac{16}{175}\right]$$

$$c_1 - \cancel{c_2} = -\frac{16}{175}$$

$$c_1 + \cancel{c_2} = \frac{16}{175}$$

$$c_1 = 0$$

$$x = 0 + \frac{16}{175}e^{-5t} - \frac{2t}{5} + \frac{3}{7}e^{2t} - \frac{13}{25}$$

$$\left[y = 0 - \frac{16}{175}e^{-5t} + \frac{4}{7}e^{2t} - \frac{3t}{5} - \frac{12}{25} \right] \text{Ans}$$

(4) solve simultaneous differential equation $\frac{dx}{dt} + 5x - 2y = t$

$\frac{dy}{dt} + 2x + y = 0$ being given that $x = 0, y = 0$ when $t = 0$

Solve

$$Dx + 5x - 2y = t$$
$$Dy + 2x + y = 0$$

$$(D+5)x - 2y = t \quad \times (D+1)$$
$$2x + (D+1)y = 0 \quad \times 2$$

$$(D^2+6D+5)x - 2(D+1)y = (D+1)t$$
$$4x + 2(D+1)y = 0$$

$$(D^2+6D+5+4)x = t+1$$
$$(D^2+6D+9)x = t+1$$

A.E $\quad m^2 + 6m + 9 = 0$

$$m = -3, -3$$

$$[\ C.F = c_1 e^{-3t} + t c_2 e^{-3t}\ ]$$

$$P.I = \frac{1}{(D^2+6D+9)}(t+1)$$

$$= \frac{1}{9\left[1 + \frac{D^2+6D}{9}\right]}(t+1)$$

$$= \frac{1}{9}\left[1 + \frac{D^2+6D}{9}\right]^{-1}(t+1)$$

$$= \frac{1}{9}\left[1 - \frac{6D}{9}\right](t+1)$$

$$= \frac{1}{9}\left(t+1 - \frac{6}{9}\right) = \frac{1}{9}\left(t + \frac{1}{3}\right)$$

$x = C.F + P.I$

$x = c_1 e^{-3t} + t c_2 e^{-3t} + \frac{1}{9}\left(t + \frac{1}{3}\right)$

$\frac{dx}{dt} = -3c_1 e^{-3t} \cancel{+ tc} - 3tc_2 e^{-3t} + c_2 e^{-3t} + \frac{1}{9}$

from equation ①

$$\frac{dx}{dt} + 5x - 2y = t$$

$$\frac{dx}{dt} + 5x = 2y + t$$

$$\frac{dx}{dt} + 5x - t = 2y$$

$$y = \frac{1}{2}\frac{dx}{dt} + \frac{5}{2}x - \frac{t}{2}$$

$$y = \frac{1}{2}\left[-3c_1 e^{-3t} - 3tc_2 e^{-3t} + c_2 e^{-3t} + \frac{1}{9}\right] + \frac{5}{2}\left[c_1 e^{-3t} + tc_2 e^{-3t}\right]$$

$$+ \frac{1}{9}\left(t + 1/3\right) - \frac{t}{2}$$

$$= \frac{-3}{2}c_1 e^{-3t} - \frac{3}{2}tc_2 e^{-3t} + \frac{1}{2}c_2 e^{-3t} + \frac{2}{9} + \frac{5}{2}c_1 e^{-3t} + \frac{5}{2}tc_2 e^{-3t}$$

$$+ \frac{1}{9}t + \frac{1}{27} - \frac{t}{2}$$

$$\left[y = c_1 e^{-3t} + c_2 t e^{-3t} + \frac{1}{2}c_2 e^{-3t} + \frac{11t}{18} + \frac{7}{27} \right]$$

$x(0) = 0$

$0 = c_1 + \frac{1}{27}$

$c_1 = -\frac{1}{27}$

$y(0) = 0$

$0 = c_1 + \frac{1}{2}c_2 + \frac{7}{27}$

$0 = -\frac{1}{27} + \frac{1}{2}c_2 + \frac{7}{27}$

$0 = \frac{1}{2}c_2 + \frac{6}{27}\ \frac{2}{9}$

$\frac{1}{2}c_2 = -\frac{2}{9}$

$\left[c_2 = -\frac{4}{9}\right]$

$$\left[x = -\frac{1}{27}e^{3t} + t\left(-\frac{2}{9}\right)e^{3t} + \frac{1}{9}\left(8t + \frac{1}{3}\right) \right] \underline{\text{Ans}}$$

$$\left[y = -\frac{1}{27}e^{3t} - \frac{4}{9}te^{3t} - \frac{5}{18}c_2 e^{3t} + \frac{11t}{18} + \frac{7}{27} \right]$$

(5) solve simultaneous differential equn

$$\frac{dx}{dt} + 2x + 4y = 1 + 4t \qquad \frac{dy}{dt} + x - y = \frac{3}{2}t^2$$

$$Dx + 2x + 4y = 1 + 4t$$
$$Dy + x - y = 3/2\, t^2$$

$$(D+2)x + 4y = (1+4t)\ (D-1)$$
$$x + (D-1)y = \frac{3}{2}t^2 \quad \times 4$$

$$(D^2 + D - 2 - 4)x = (3 - 4t) - 6t^2$$
$$(D^2 + D - 6)x = (3 - 4t) - 6t^2$$
$$(D^2 + D - 6)x = -6t^2 - 4t + 3$$

A.E $\quad m^2 + m - 6 = 0$

$$m = 2, -3$$

$$C.F = c_1 e^{2t} + c_2 e^{-3t}$$

$$P.I = \frac{1}{D^2 + D - 6}(-6t^2 - 4t + 3)$$

$$= \frac{1}{-6\left[1 - \frac{D^2 + D}{6}\right]}\left[-6t^2 - 4t + 3\right)$$

$$= -\frac{1}{6}\left[1 - \frac{D^2 + D}{6}\right]^{-1}\left[-6t^2 - 4t + 3\right]$$

$$= -\frac{1}{6}\left[1 + \frac{D^2 + D}{6} + \frac{D^2}{36}\right]\left[-6t^2 - 4t + 3\right]$$

$$= -\frac{1}{6}\left[1 + \frac{D^2}{6} + \frac{D}{6} + \frac{D^2}{36}\right]\left[-6t^2 - 4t + 3\right]$$

$$= -\frac{1}{6}\left[1+\frac{D^2}{6}+\frac{D}{6}+\frac{D^2}{36}\right](3-4t-6t)$$

$$= -\frac{1}{6}\left[3-4t-6t^2-2-2/3(1+3t)-\frac{1}{3}\right]$$

$$= -\frac{1}{6}\left[1-4t-6t^2-\frac{2}{3}\cdot 2t-\frac{1}{3}\right]$$

$$= -1/6\left[-6t-6t^2\right] = t+t^2$$

$P.I = t+t^2$

$x = c_1e^{2t}+c_2e^{-3t}+t+t^2$

$\frac{dx}{dt} = 2c_1e^{2t}-3c_2e^{-3t}+2t+1$

(7) Solve the simultaneous diff equation -

$$\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 4x = y \quad \text{(I)} \qquad \frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = 25x + 16e^t \quad \text{(II)}$$

$$(D^2 - 4D + 4)x - y = 0 \qquad \times (D^2 + 4D + 4)$$

$$-25x + (D^2 + 4D + 4)y = 16e^t \times 1$$

$$(D^4 - 8D^2 - 9)x = 16e^t$$

A.E $\quad M^4 - 8M^2 - 9 = 0$

let

$M^2 = K \qquad K^2 - 8K - 9 = 0$

$K = 9, -1$

$m^2 = 9 \qquad m = 3, -3$

$m^2 = -1 \quad m = \pm i$

$$C.F = c_1 e^{3t} + c_2 e^{-3t} + (c_3 \cos t + c_4 \sin t)$$

$$P.I = \frac{1}{D^4 - 8D^2 - 9} 16e^t$$

$D = a = 1$

$$= \frac{1}{1 - 8 - 9} 16e^t$$

$P.I = -e^t$

$$\left[\ x = c_1 e^{3t} + c_2 e^{-3t} + c_3 \cos t + c_4 \sin t - e^t\ \right]$$

$$\frac{dx}{dt} = 3c_1 e^{3t} - 3c_2 e^{-3t} - c_3 \sin t + c_4 \cos t - e^t$$

$$\frac{d^2x}{dt^2} = 9c_1 e^{3t} + 9c_2 e^{-3t} - c_3 \cos t - c_4 \sin t - e^t$$

from (I)

$y = 9c_1e^{3t} + 9c_2e^{-3t} - c_3\cos t - c_4\sin t - e^t$
$\quad -12c_1e^{3t} + 12c_2e^{-3t} + 4c_3\sin t - 4c_4\cos t + 4e^t$
$\quad + 4c_1e^{3t} + 4c_2e^{-3t} + 4c_3\cos t + 4c_4\cos t - 4e^t$

$$\left[y = c_1e^{3t} + 25c_2e^{-3t} + 4c_3\sin t + 3c_3\cos t - 4c_4\cos t + 3c_4\sin t - e^t \right]$$

(2) $\frac{d^2x}{dt^2} + 4x + 5y = t^2$ — (I) $\quad \frac{d^2y}{dt^2} + 5x + 4y = t + 1$ — (II)

$(D^2+4)x + 5y = t^2 \quad \times (D^2+4)$

$25x + (D^2+4)y = t + 1 \quad \times 5$

$(D^4 + 8D^2 + 16)x - 25x = 4t^2 - 5t - 3$

$(D^4 + 8D^2 - 9)x = 4t^2 - 5t - 3$

A.E

$M^4 + 8M^2 - 9 = 0$

let

$M^2 = K \qquad K^2 + 8K - 9 = 0$

$K = 1, -9$

$M^2 = 1 \Rightarrow M = \pm 1$

$M^2 = -9 \Rightarrow M = \pm 3i$

$C.F = c_1e^{t} + c_2e^{-t} + (c_3\cos 3t + c_4\sin 3t)$

$$P.I = \frac{1}{D^4 + 8D^2 - 9}(4t^2 - 5t - 3)$$

$$= \frac{1}{-9\left[1 - \frac{D^4 + 8D^2}{9}\right]}[4t^2 - 5t - 3]$$

$$= -\frac{1}{9}\left[1 - \frac{D^4 + 8D^2}{9}\right]^{-1} \left[4t^2 - 5t - 3\right]$$

$$= -\frac{1}{9}\left[1 + \frac{D^4 + 8D^2}{9}\right]\left[4t^2 - 5t - 3\right]$$

$$= -\frac{1}{9}\left[1 + \frac{8D^2}{9}\right]\left[4t^2 - 5t - 3\right]$$

$$= -\frac{1}{9}\left[4t^2 - 5t - 3 + \frac{8 \times 8}{9}\right]\left[9\right.$$

Q.6. Solve the following simultaneous equation

$$\frac{d^2x}{dt^2} + y = \sin t \quad , \quad \frac{d^2y}{dt^2} + x = \cos t$$

$$D^2x + y = \sin t \quad \times D^2 \qquad -(1)$$

$$x + D^2y = \cos t \quad \times 1 \qquad -(11)$$

$$D^4x + D^2y = \cancel{\cos t} \; -\sin t$$

$$x + D^2y = \cos t$$

$$- \quad - \qquad -$$

$$(D^4 - 1)x = 0 - (\sin t + \cos t)$$

$$m^4 - 1 = 0$$

let $m^2 = k$ $\qquad k^2 - 1 = 0$

$$k = \pm 1, \pm i$$

$$m = \pm 1, \pm i$$

$$C.F = c_1e^t + c_2e^{-t} + [c_3\cos t + c_4\sin t]$$

$$P.I = \frac{1}{D^4-1}\sin t - \frac{1}{D^4-1}\cos t$$

$$D^2 = -a^2 = -1 \qquad D^2 = -a^2 = -1$$

$$P.I = \frac{1}{1-1}\sin t - \frac{1}{1-1}\cos t$$

case fail case fail.

$$= \frac{t}{4D^3}\sin t - \frac{t}{4D^3}\cos t$$

$$= \frac{t}{4D^2 \cdot D}\sin t - \frac{t}{4D^2 \cdot D}\cos t$$

$\cancel{a^2 = D}$ $D^2 = -a^2 = -1$ $D^2 = -1$

$$= \frac{t}{-4D}\sin t + \frac{t}{4D}\cos t$$

$$= \frac{t\sin t}{4D} + \frac{t}{4D}\cos t$$

$$\left[P.I = -\frac{t\cos t}{4} + \frac{t\sin t}{4}\right]$$

$$x = C.F + P.I$$

$$x = \left[c_1e^t + c_2e^{-t} + [c_3\cos t + c_4\sin t] - \frac{t\cos t}{4} + \frac{t\sin t}{4}\right]$$

$\cancel{\frac{dx}{dt} = c_1e^t +}$

$$\frac{dx}{dt} = c_1e^t - c_2e^{-t} - c_3\sin t + c_4\cos t - \left[\frac{t(-\sin t) + \cos t}{4}\right.$$

$$\left. + \frac{t\cos t}{4} + \frac{\sin t}{4}\right]$$

$$\frac{dx}{dt} = c_1e^t - c_2e^{-t} - c_3\sin t + c_4\cos t + \frac{t}{4}\sin t - \frac{1}{4}\cos t + \frac{t}{4}\cos t$$

$$+ \frac{1}{4}\sin t$$

$$\frac{d^2x}{dt^2} = c_1e^t + c_2e^{-t} - c_3\cos t - c_4\sin t + \frac{t}{4}\cos t + \frac{1}{4}\sin t + \frac{1}{4}\sin t$$
$$- \frac{t}{4}\sin t + \frac{1}{4}\cos t + \frac{1}{4}\cos t$$

equation from (1)

$$y = \sin t - \frac{d^2x}{dt^2}$$

$$\left[\, y = -c_1e^t - c_2e^{-t} + c_3\cos t + c_4\sin t - \frac{t}{4}\cos t + \frac{t}{4}\sin t - \frac{1}{2}\cos t + \frac{1}{2}\sin t \,\right]$$

(10) solve $\frac{d^2x}{dt^2} + \frac{dy}{dt} + 3x = e^{-t}$, $\frac{d^2y}{dt^2} - 4\frac{dx}{dt} + 3y = \sin 2t$

$$(D^2+3)x + Dy = e^{-t} \qquad \times (D^2+3)$$
$$-4Dx + (D^2+3)y = \sin 2t \qquad \times D$$

$$(D^2+3)^2x + D(D^2+3)y = e^{-t} + 3e^{-t}$$
$$-4D^2x + D(D^2+3)y = 2\cos 2t$$
$$+ \qquad - \qquad -$$

$$[(D^2+3)^2 + 4D^2]x = e^{-t} + 3e^{-t} - 2\cos 2t$$
$$[D^4 + 10D^2 + 9]x = 4e^{-t} - 2\cos 2t$$

A.E $\quad m^4 + 10m^2 + 9 = 0$

let $m^2 = K$ $\quad K^2 + 10K + 9 = 0$

$$K = -1, -9$$
$$m = \pm i, \pm 3i$$

$$[\, C.F = c_1\cos t + c_2\sin t + c_3\cos 3t + c_4\sin 3t \,]$$

$$P.I = \frac{1}{D^4+10D^2+9}4e^{-t} - \frac{1}{D^4+10D^2+9}2\cos 2t$$

$$D = a = -1 \qquad D^2 = a^2 = -4$$

$$P.I = \frac{4e^{-t}}{1+10+9} - \frac{1}{16-40+9}2\cos 2t$$

$$\left[P.I = \frac{\cancel{4}e^{-t}}{\cancel{20}5} + \frac{2}{15}\cos 2t \right]$$

$$\left[x = c_1\cos t + c_2\sin t + c_3\cos 3t + c_4\cos 3t + \frac{e^{-t}}{5} + \frac{2}{15}\cos 2t \right]$$

$$\frac{dx}{dt} = -c_1\sin t + c_2\cos t - 3c_3\sin 3t - 5_4\cos 3t - \frac{e^{-t}}{5} - \frac{4}{15}\cos 2t$$

$$\frac{d^2x}{dt^2} = -c_1\cos t - c_2\sin t - 9c_3\cos 3t - 9c_4\sin 3t + \frac{e^{-t}}{5} - \frac{8}{15}\cos 2t$$

from equation ①

$$\frac{dy}{dt} = e^{-t} - 3x - \frac{d^2x}{dt^2}$$

$$= e^{-t} - 3c_1\cos t - 3c_2\sin t - 3c_3\cos 3t - 3c_4\sin 3t - \frac{3}{5}e^{-t} - \frac{2}{5}\cos 2t$$
$$c_1\cos t + c_2\sin t + 9c_3\cos 3t + 9c_4\sin 3t - \frac{e^{-t}}{5} + \frac{8}{15}\cos 2t$$

$$\frac{dy}{dt} = -2c_1\cos t - 2c_1\sin t + 6c_3\cos 3t + 6c_4\sin 3t + \frac{1}{5}e^{-t} + \frac{2}{15}\cos 2t$$

$$\left[y = -2c_1\sin t + 2c_1\cos t + 2c_3\sin 3t - 2c_4\cos 3t - \frac{1}{5}e^{-t} + \frac{1}{15}\sin 2t \right]$$

Q 9. Solve the following simultaneous eqn $\frac{dx}{dt} = -wy$ $\frac{dy}{dt} = wx$ also show point (x,y) lies on a circle.

$$Dx + wy = 0 \quad \times D \quad (1)$$
$$-wx + Dy = 0 \quad \times w$$

$$D^2x + Dwy = 0$$
$$-w^2x + Dwy = 0$$
$$+ \qquad -$$

$$(D^2 + w^2)x = 0$$

A.E $\quad m^2 + w^2 = 0$

$\quad m = \pm wi$

$C.F = c_1\cos wt + c_2 \sin wt$

$[P.I = 0]$

$x = c_1\cos wt + c_2\sin wt$

$\frac{dx}{dt} = -wc_1\sin wt + wc_2 \cos wt$

from eq. (1)

$$y = \frac{-1}{w}\frac{dx}{dt}$$

$$\left[y = c_1\sin wt - c_2\cos wt \right]$$

$x^2+y^2 = c_1^2\cos^2 wt + c_2^2\sin^2 wt + 2c_1\cos wt\, c_2\sin wt + c_1^2\sin^2 wt$
$+ c_2^2\cos^2 wt - 2c_1c_2\sin wt\cos wt$

$x^2+y^2 = c_1^2+c_2^2 \qquad$ let $c_1^2 + c_2^2 = a^2$

$$\left[x^2+y^2 = a^2 \right]$$