Unit – II LAPLACE TRANSFORM & Inverse Laplace Transform:

Short Answer Questions:

1. Find the Laplace transform of $f(t)=t^4e^{2t}$ **[2023-24]**
2. **Explain the first shifting property of Laplace transform with example [2022-23].**
3. Find the inverse Laplace transform of $F(s)=\frac{1}{s^2+2s+2}$ **[2022-23].**

(i) Find the inverse Laplace transform of (a) $\frac{s+8}{s^2+4s+5}$ **[2017-18]. (b)** $\frac{1}{s^2-3s+3}$ **[2012-13].**

4. (a) If $L\{f(\sqrt{t})\}=\frac{e^{-1/s}}{s}$, find $L\{e^{-t}f(3\sqrt{t})\}$ **[2017-18]. (b)** If $L\{f(t)\}=\frac{e^{-\frac{1}{s}}}{s}$ then find $L\{e^{-t}f(3t)\}$ **[2013-14]**
5. P.T. $L\{e^{at}f(t)\}=F(s-a)$ **[2016-17]**

OR

If Laplace transform of $f(t)$ is $F(s)$, then show that the Laplace transform of . $\{e^{at}f(t)\}$ is $F(s-a)$ where a is any real number. **[2012-13]**

6. Find the inverse Laplace transform of $F(s)=\frac{s}{2S^2+8}$**[2015-16]**
7. Find the Laplace transform of $\frac{\sin at}{t}$ **[2016-17].**
8. Find the inverse Laplace transform of $\frac{e^{-\pi s}}{(s^2+1)}$ **[2014-15].**
9. Find the laplace transform of $\int_0^t\int_0^t \sin u\, du du$ **[2014-15]**
10. Find the laplace transform of unit step function $u(t-a)$. **[2015-16]**
11. Find the laplace transform of unit step function $u(t)$.**[2013-14]**

LAPLACE TRANSFORM:

1. Find the Laplace transform

(i) $\int_0^t \sin 3t \cos h2t\ dt$

(ii) $t\,e^{-t}\sin 2t$. **[2003]**

(iii) $t^2 e^t \sin 4t$.**[2001]**

(iv) $\frac{1-\cos t}{t}$**[2014-15]**

(v) $\frac{e^{at}-\cos bt}{t}$

(vi) $\frac{\cos at-\cos bt}{t}$**[2016-17]**

(vii) $\int_0^t e^{-t}\cos t\ dt$ **[2021-22]**

(viii) $\int_0^t e^{-2t}t^2\cos t\ dt$

(ix) $f(t)=\frac{1-\cos t}{t^2}$**[2023-24]**

(x) $\cos hat \cos bt$**[2015-16]** **(xiii)** $\int_0^t \frac{\cos 2t-\cos 3t}{t}\,dt$**[2013-14]** **(xiv)**

2. Evaluate the following integrals

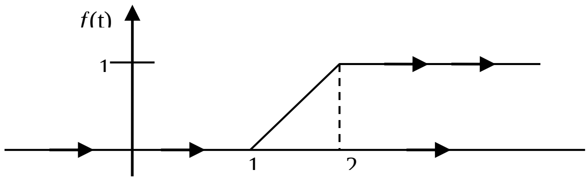
(i) $\int_0^\infty \frac{e^{-3t}\sin t}{t}\,dt$**[2014-15]** **(ii)** $\int_0^\infty \frac{e^{-at}-e^{-bt}}{t}\,dt$ **(iii)** Show that $\int_0^\infty \frac{\cos 6t-\cos 4t}{t}\,dt=\log\frac{2}{3}$. **[2011-12]**

(iv) $\int_0^\infty e^{-2t}t^2\sin 3t\ dt$**[2014-15]** **(v)** $\int_0^\infty \frac{e^{-3t}\sin t}{t}\,dt$**[2012-13]** **(vi)** Show that $\int_0^\infty\int_0^t \frac{e^{-t}\sin u}{u}\,du dt=\frac{\pi}{4}$ **[2010-11]**

3. Find the Laplace transform of the function $f(x)=x^3\sin x$. Hence prove that $\int_0^\infty e^{-x}x^3\sin x\ dx=0$. **[2022-23]**

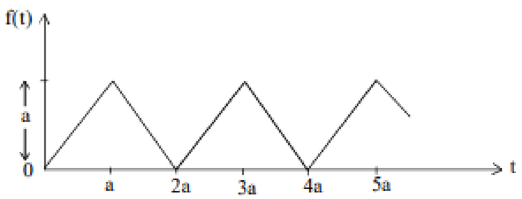
UNIT STEP FUNCTION:

1. Find the Laplace transform of **(i)** $(t-1)^2U(t-1)$ **(ii)** $\sin t\ U(t-\pi)$**(iii)** $e^{-3t}U(t-2)$
2. Express the following function $f(t)=\begin{cases} t-1, & 1<t<2\\ 3-t, & 2<t<3\end{cases}$ in terms of unit step function & hence find its Laplace transform.**[2014-15]**
3. State second shifting theorem for Laplace transform and hence find the L.T. of $f(t)=\begin{cases} e^{t-a} & , t>a\\ 0 & , t<a\end{cases}$.**[2011-12]**
4. Express the following function in terms of unit step function and find its Laplace transform. **[UPTU 2002]**



PERIODIC FUNCTION:

1. Find the Laplace transform of the periodic function shown in figure.**[2013-14]**



2. Draw the graph and find the Laplace transform of the triangular wave function of period 2c given by $f(t) = \begin{cases} t & , \quad 0 < t \le \pi \\ 2\pi - t & , \quad c < t < 2\pi \end{cases}$. **[2017-18]**

3. Find the Laplace transform of square wave function of period a defined as $f(t) = \begin{cases} 1 & , \quad 0 \le t \le a/2 \\ -1 & , \quad a/2 < t < a \end{cases}$.**[2004]**

4. A periodic function is defined by $f(t) = \begin{cases} \sin \omega t & 0 \le t \le \pi/\omega \\ 0 & \pi/\omega \le t \le 2\pi/\omega \end{cases}$. Find Laplace transform of $f(t)$.**[2010-11]**

5. Determine the Laplace transform of the periodic function defined by the triangular wave function of period 2a $f(t) = \begin{cases} \frac{t}{a} & \text{for} \quad 0 \le t \le a \\ \frac{2a-t}{a} & \text{for} \quad a \le t \le 2a \end{cases}$. **[2021-22]**

Inverse Laplace Transform:

1. Find the inverse Laplace transform of the following
   - **(i)** $\frac{e^{-2p}}{p^2}$**[2021-22]**
   - **(ii)** $\frac{3}{s^2+2s-8}$**[2013-14]**
   - **(iii)** $\log\frac{s+a}{s+b}$**[2003]**
   - **(iv)** $\frac{5S+3}{(S-1)(S^2+2S+5)}$**[2005]**
   - **(v)** $\frac{1}{(s+3)^4}$**[2014-15]**
   - **(vi)** $\frac{e^{-s}}{\sqrt{s+1}}$**[2014-15]**
   - **(vii)** $\cot^{-1}\left(\frac{s+3}{2}\right)$**[2012-13]**

2. Find the function whose Laplace transform is $F(s)=\log\frac{(s^2+1)}{s(s+1)}$. **[2013-14]**

3. Find the function whose Laplace transform is $\log\left(1+\frac{1}{s^2}\right)$ **[2014-15].**

CONVOLUTION THEOREM:

1. Use convolution theorem to evaluate $L^{-1}\left\{\frac{S}{(S^2+1)(S^2+4)}\right\}$**[2012-13]**

2. Use convolution theorem to evaluate$L^{-1}\frac{1}{(p^2+a^2)^2}$ **[2014-15]** OR $L^{-1}\frac{1}{(s^2+a^2)^2}$ **[2023-24]**

3. Use convolution theorem to evaluate $L^{-1}\frac{s}{(s^2+a^2)^3}$ **[2011-12].**

4. Use convolution theorem to evaluate $L^{-1}\frac{16}{(s-2)(s+2)^2}$ **[2010-11] [2013-14].**

5. Use convolution theorem to evaluate $L^{-1}\left[\frac{S^2}{(S^2+a^2)(S^2+b^2)}\right]$**[2003-04] [2005-06] [2017-18]**

6. Using convolution theorem, prove that $L^{-1}\left\{\frac{1}{S^3(S^2+1)}\right\} = \frac{t^2}{2} + \cos t - 1.$ **[2004-05]**

7. State convolution theorem of Laplace transform and using it find $L^{-1}\left\{\frac{1}{(S^2+4)(S+2)}\right\}$. **[2015-16]**

8. State convolution theorem of Laplace transform, hence find inverse Laplace transform of $\frac{1}{S^2(S+1)^2}$ **[2022-23]**

Application of Laplace Transform:

1. Solve using Laplace transform y''+ 4y'+ 4y= $6e^{-t}$ y(0) = -2, y'(0) = 8. **[2006-07] [20023-24]**

2. Using convolution solve the initial value problem $\frac{d^2y}{dt^2} + 9y = \sin 3t$, given y(0) = 0, $\frac{dy}{dt}$= 0 when t=0.**[2011-12]**

3. Using Laplace transform solve the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}\sin t$, x(0) = 0, x'(0) = 1.**[2010-11]**

4. Find the soln. of the differential equation $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = e^t$ with the initial conditions y(t) = 0, $\frac{dy}{dt}$= 1.when t=0 using Laplace transform. **[2012-13]**

5. Using Laplace transform solve y'' + 2y' + y = te^{-t} under the conditions y(0) =+1, y'(0) = -2.**[2014-15]**

6. Apply Laplace transform to solve $\frac{d^2y}{dt^2} + y = t\cos 2t, t > 0$ given that y = $\frac{dy}{dt}$ = 0 for t = 0. **[2015-16]**

7. Using L.T. to solve the differential equation $\frac{d^2y}{dt^2} + 9y = \cos 2t$ where y(0) = 1, y'($\frac{\pi}{2}$) = -1. **[2016-17]**

8. Using Laplace transform solve the differential equation $\frac{d^2y}{dx^2} + y = 6\cos 2x$, y(0) = 3 and y'(0) = 1 **[2022-23]**

9. Solve the following differential equation using Laplace transform $\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - y = t^2e^t$,where y(0) = 1, y'(0) = 0 and y''(0) = -2. **[2017-18]**

10. Use Laplace transform to solve $\frac{dx}{dt} + y = \sin t; \frac{dy}{dt} + x = \cos t$ given that x = 2, y = 0 at t = 0. **[2003-04]**